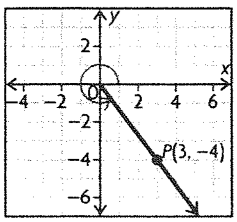
CHAPTER 6
Trigonometric Functions

Getting Started, p. 314

1. a) 28°
b) 360° − 28° = 332°
2. a)

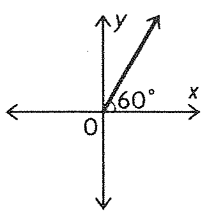


Side opposite: −4
Side adjacent: 3
Hypotenuse: $h^2 = 3^2 + 4^2$
$h^2 = 25$
$h = 5$

$\sin\theta = -\frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = -\frac{4}{3},$

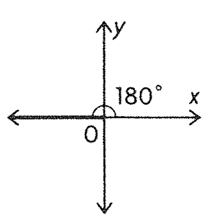
$\csc\theta = -\frac{5}{4}, \sec\theta = \frac{5}{3}, \cot\theta = -\frac{3}{4},$

b) $\theta = \sin^{-1}\left(-\frac{4}{5}\right)$
$\theta \doteq 307°$
The principal angle is 360° − 53° = 307°
3. a)



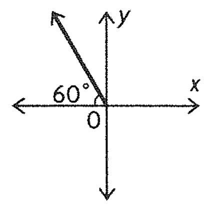
$\sin 60° = \frac{\sqrt{3}}{2}$

b)



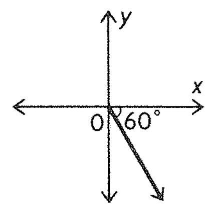
$\tan 180° = \frac{0}{1} = 0$

c)



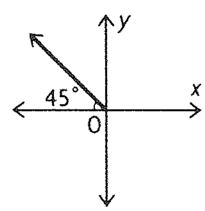
$\sin 120° = \frac{\sqrt{3}}{2}$

d)



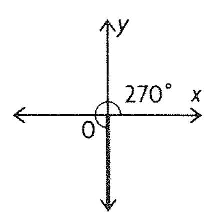
$\cos 300° = \frac{1}{2}$

e)



$\sec 135° = -\frac{\sqrt{2}}{1} = -\sqrt{2}$

f)

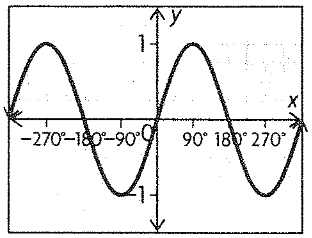


$\csc 270° = \frac{1}{-1} = -1$

4. a) Since cosine is positive in the first and fourth quadrants, $\theta = 60°, 300°$
b) Since tangent is positive in the first and third quadrants, $\theta = 30°, 210°$
c) Since tangent is positive in the first and third quadrants, $\theta = 45°, 225°$
d) Cosine equals −1 at $\theta = 180°$

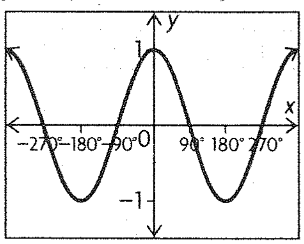
e) Cotangent equals -1 at $\theta = 135°, 315°$
f) Sine equals 1 at $\theta = 90°$

5. a)



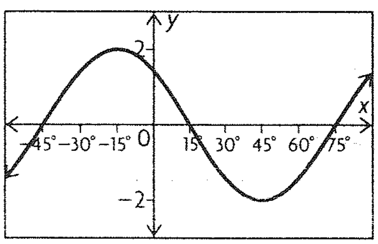
period $= 360°$; amplitude $= 1$; $y = 0$;
$R = \{y \in \mathbf{R} \mid -1 \le y \le 1\}$

b)

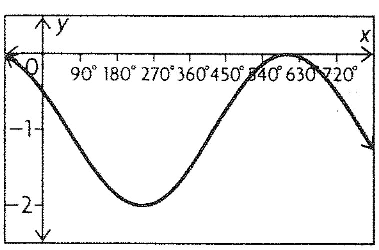


period $= 360°$; amplitude $= 1$; $y = 0$;
$R = \{y \in \mathbf{R} \mid -1 \le y \le 1\}$

6. a) period $= \dfrac{2(180)}{3} = 120$;
$y = 0$; 45° to the left; amplitude $= 2$



b) period $= \dfrac{2(180)}{\frac{1}{2}} = 360(2) = 720°$;
$y = -1$; 60° to the right; amplitude $= 1$



7. a is the amplitude, which determines how far above and below the axis of the curve of the function rises and falls; k defines the period of the function, which is how often the function repeats itself; d is the horizontal shift, which shifts the function to the right or the left; and c is the vertical shift of the function.

6.1 Radian Measure, pp. 320–322

1. a) π radians;

$$\cancel{\pi \text{ radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = 180°$$

b) $\dfrac{\pi}{2}$ radians;

$$\frac{\pi}{2}\cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = 90°$$

c) $-\pi$ radians;

$$-\cancel{\pi \text{ radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = -180° = 180°$$

d) $-\dfrac{3\pi}{2}$ radians $= \dfrac{\pi}{2}$ radians;

$$-\frac{3\pi}{2}\cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = -270°$$

e) -2π radians;

$$-2\cancel{\pi \text{ radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = -360°$$

f) $\dfrac{3\pi}{2}$ radians;

$$\frac{3\pi}{2}\cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = 270°$$

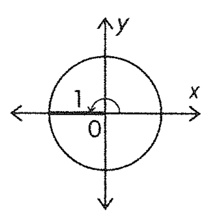
g) $-\dfrac{4\pi}{3}$ radians;

$$-\frac{4\pi}{3}\cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = -240°$$

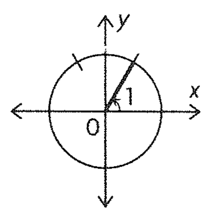
h) $\dfrac{2\pi}{3}$ radians;

$$\frac{2\pi}{3}\cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi \text{ radians}}}\right) = 120°$$

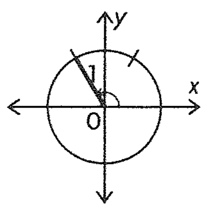
2. a)



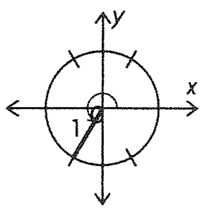
b)



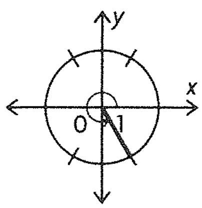
c)



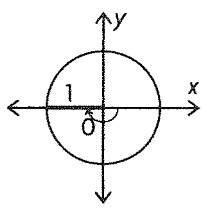
d)



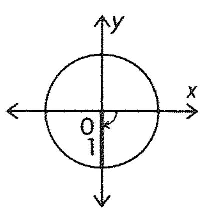
e)



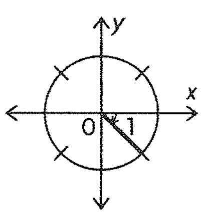
f)



g)



h)



3. a) $75° = 75° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{5\pi}{12} \text{ radians}$

b) $200° = 200° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{10\pi}{9} \text{ radians}$

c) $400° = 400° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{20\pi}{9} \text{ radians}$

d) $320° = 320° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{16\pi}{9} \text{ radians}$

4. a) $\dfrac{5\pi}{3} = \dfrac{5\pi}{3} \times \left(\dfrac{180°}{\pi \text{ radians}}\right) = 300°$

b) $0.3\pi = 0.3\pi \times \left(\dfrac{180°}{\pi \text{ radians}}\right) = 54°$

c) $3 = 3 \times \left(\dfrac{180°}{\pi \text{ radians}}\right) = 171.89°$

d) $\dfrac{11\pi}{4} = \dfrac{11\pi}{4} \times \left(\dfrac{180°}{\pi \text{ radians}}\right) = 495°$

5. a) $5 = \dfrac{x°}{360°}(2\pi)(2.5)$

$1800 = (x)(2\pi)(2.5)$

$114.6° = x$

$114.6° = 114.6° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = 2 \text{ radians}$

b) $200° = 200° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{10\pi}{9} \text{ radians}$

$$x = \frac{\frac{10\pi}{9}}{2\pi}(2\pi)(2.5)$$

$$x = \frac{5}{9}(2\pi)(2.5)$$

$$x = \frac{25\pi}{9} \text{ cm}$$

6. a) $3.5 = 3.5 \times \left(\dfrac{180°}{\pi \text{ radians}}\right) = 200.5°$

$$x = \frac{200.5°}{360°}(2\pi)(8)$$

$$x = 28 \text{ cm}$$

b) $300° = 300° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{5\pi}{3} \text{ radians}$

$$x = \frac{\frac{5\pi}{3}}{2\pi}(2\pi)(8)$$

$$x = \frac{5}{6}(2\pi)(8)$$

$$x = \frac{40\pi}{3} \text{ cm}$$

7. a) $90° = 90° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{\pi}{2} \text{ radians}$

b) $270° = 270° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) = \dfrac{3\pi}{2} \text{ radians}$

c) $-180° = -180° \times \left(\dfrac{\pi \text{ radians}}{180°}\right)$

$= \boxed{-\pi} = \pi \text{ radians}$

d) $45° = 45° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{4}\text{radians}$

e) $-135° = -135° \times \left(\frac{\pi \text{ radians}}{180°}\right)$

$= \boxed{-\frac{3\pi}{4}} = \frac{5\pi}{4}\text{radians}$

f) $60° = 60° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{3}\text{radians}$

g) $240° = 240° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{4\pi}{3}\text{radians}$

h) $-120° = -120° \times \left(\frac{\pi \text{ radians}}{180°}\right)$

$= \boxed{-\frac{2\pi}{3}} = \frac{4\pi}{3}\text{radians}$

8. a) $\frac{2\pi}{3} = \frac{2\pi}{3} \times \left(\frac{180°}{\pi \text{ radians}}\right) = 120°$

b) $-\frac{5\pi}{3} = -\frac{5\pi}{3} \times \left(\frac{180°}{\pi \text{ radians}}\right) = -300° = 60°$

c) $\frac{\pi}{4} = \frac{\pi}{4} \times \left(\frac{180°}{\pi \text{ radians}}\right) = 45°$

d) $-\frac{3\pi}{4} = -\frac{3\pi}{4} \times \left(\frac{180°}{\pi \text{ radians}}\right) = -135° = 225°$

e) $\frac{7\pi}{6} = \frac{7\pi}{6} \times \left(\frac{180°}{\pi \text{ radians}}\right) = 210°$

f) $-\frac{3\pi}{2} = -\frac{3\pi}{2} \times \left(\frac{180°}{\pi \text{ radians}}\right) = -270° = 90°$

g) $\frac{11\pi}{6} = \frac{11\pi}{6} \times \left(\frac{180°}{\pi \text{ radians}}\right) = 330°$

h) $-\frac{9\pi}{2} = -\frac{9\pi}{2} \times \left(\frac{180°}{\pi \text{ radians}}\right) = -810°$

$= -90° = 270°$

9. a) $x = \frac{\frac{19\pi}{20}}{2\pi}(2\pi)(65)$

$x = \frac{19}{40}(2\pi)(65)$

$x = \frac{2470\pi}{40}$

$x = \frac{247\pi}{4}\text{ m}$

b) $1.25 = 1.25 \times \left(\frac{180°}{\pi \text{ radians}}\right) = 143.2°$

$x = \frac{143.2°}{360°}(2\pi)(65)$

$x = 162.5\text{ m}$ $x = 65(1.25)$

$= 81.25\text{m}$

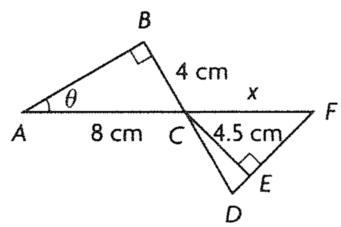
c) $150° = 150° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{5\pi}{6}\text{radians}$

$x = \frac{\frac{5\pi}{6}}{2\pi}(2\pi)(65)$

$x = \frac{5}{12}(2\pi)(65)$

$x = \frac{325\pi}{6}\text{ cm}$

10.



$\sin\theta = \frac{4}{8}$ or $\frac{1}{2}$, so $\theta = \frac{\pi}{6}$.

$\angle BCA = \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}$

Because they are vertical angles, $\angle BCA = \angle DCF$.

$\angle DCF = \angle DCE + \angle ECF$

$\frac{\pi}{3} = \frac{\pi}{12} + \angle ECF$

$\frac{\pi}{3} - \frac{\pi}{12} = \angle ECF$

$\frac{\pi}{4} = \angle ECF$

Since $\angle ECF = \frac{\pi}{4}$, $x = \sqrt{2}(CE)$

$= 4.50\sqrt{2}\text{ cm}$

11. a) It rotates 4 times per min. So it rotates once every 15 seconds.

$\omega = \frac{2\pi \text{ radians}}{15\text{ s}} \doteq 0.418\,88\text{ radians/s}$

b) Radius = 3 m

Revolutions, $n = (4\text{ rev/min})(5\text{ min}) = 20\text{ rev}$

distance travelled $= 20(2\pi)(3) \doteq 377.0$ m

12. a) $\omega = 1.2\pi\text{ rad/s}(60\text{ s/min})$

$= 72\pi\text{ rad/min}$

$72\cancel{\pi}\ \cancel{\text{rad}}/\text{min}\left(\frac{180°}{\cancel{\pi}\ \cancel{\text{radians}}}\right) = 12\,960°$

In one minute, the wheel rotates 12 960°. So,

Revolutions, $n = 12\,960° \div 360° = 36$

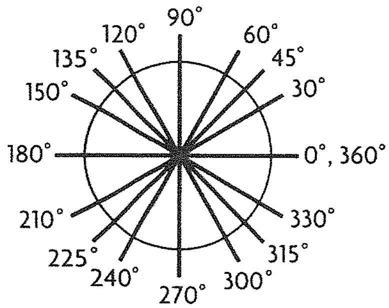
b) The wheel travels $(9.6\pi)(6) = 57.6\pi$ metres in a minute.

$57.6\pi = 36(2\pi)(r)$

$0.8\text{ m} = r$

13. a) The angular velocity of piece A is equal to piece B because they rotate at the same speed around the centre.
b) The velocity of piece A is greater than piece B because the radius of A is greater than the radius of B.
c) The percentage would stay the same.

14.



$$0° = 0° \times \left(\frac{\pi \text{ radians}}{180°}\right) = 0 \text{ radians};$$

$$30° = 30° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{6} \text{ radians};$$

$$45° = 45° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{4} \text{ radians};$$

$$60° = 60° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{3} \text{ radians};$$

$$90° = 90° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{2} \text{ radians};$$

$$120° = 120° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{2\pi}{3} \text{ radians};$$

$$135° = 135° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{3\pi}{4} \text{ radians};$$

$$150° = 150° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{5\pi}{6} \text{ radians};$$

$$180° = 180° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \pi \text{ radians};$$

$$210° = 210° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{7\pi}{6} \text{ radians};$$

$$225° = 225° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{5\pi}{4} \text{ radians};$$

$$240° = 240° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{4\pi}{3} \text{ radians};$$

$$270° = 270° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{3\pi}{2} \text{ radians};$$

$$300° = 300° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{5\pi}{3} \text{ radians};$$

$$315° = 315° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{7\pi}{4} \text{ radians};$$

$$330° = 330° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{11\pi}{6} \text{ radians};$$

$$360° = 360° \times \left(\frac{\pi \text{ radians}}{180°}\right) = 2\pi \text{ radians}$$

15. Circle A: $\frac{\frac{\pi}{6}}{2\pi}(2\pi)(15) \doteq 7.85$ cm

Circle B: $\frac{\frac{\pi}{7}}{2\pi}(2\pi)(17) \doteq 7.62$ cm

Circle C: $\frac{\frac{\pi}{5}}{2\pi}(2\pi)(14) \doteq 8.80$ cm

So, from smallest to largest, the order of the arcs would be Circle B, Circle A, and Circle C.

16. $C = 2\pi r = 2\pi(32) = 64\pi$ cm

$$64\pi \text{ cm} \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \times \left(\frac{1 \text{ km}}{1000 \text{ m}}\right) = 0.000\,64\pi \text{ km}$$

$$\text{Revolutions} = \frac{675 \text{ km}}{0.000\,64\pi \text{ km}} \doteq \cancel{1\,054\,687.5} \ 335717.5$$

$$6 \text{ hr } 45 \text{ min} = \overset{405}{\cancel{765}} \text{ min} \times \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \cancel{45\,900 \text{ s}} \ 24300 \text{ s}$$

$$\text{Rev/sec} = \frac{\cancel{1\,054\,687.5 \text{ rev}}}{\cancel{45\,900 \text{ s}}} \doteq \cancel{23 \text{ rev/s}} \doteq \frac{335717.5}{24300} \doteq 13.82$$

$$\overset{13.82}{\cancel{23}} \text{ rev} \times 360° = \cancel{8280°/\text{s}} \ 4975.2°/\text{s}$$

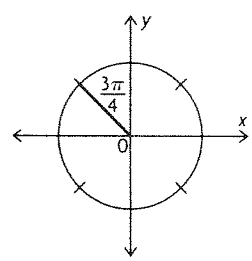
$$\overset{4975.2°/\text{s}}{\cancel{8280°/\text{s}}} = \overset{4975.2°/\text{s}}{\cancel{8280°/\text{s}}} \times \left(\frac{\pi \text{ radians}}{180°}\right)$$

$\doteq \cancel{144.5}$ radians/s

86.81

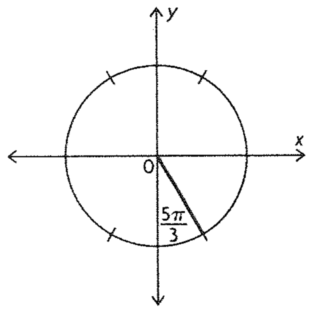
6.2 Radian Measure and Angles on the Cartesian Plane, pp. 330–332

1. a)



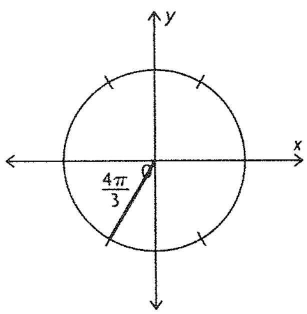
$\frac{3\pi}{4}$ is located in the second quadrant. The related angle is $\frac{\pi}{4}$, and sine is positive in the second quadrant.

b)



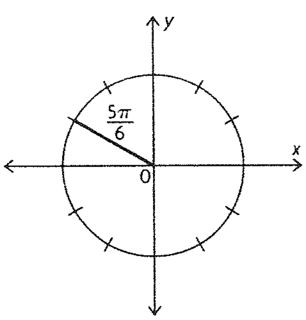
$\frac{5\pi}{3}$ is located in the fourth quadrant. The related angle is $\frac{\pi}{3}$, and cosine is positive in the fourth quadrant.

c)



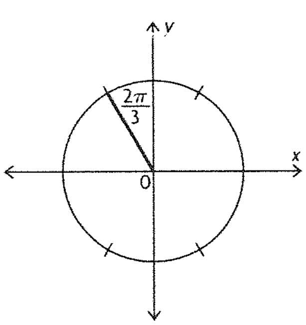
$\frac{4\pi}{3}$ is located in the third quadrant. The related angle is $\frac{\pi}{3}$, and tangent is positive in the third quadrant.

d)



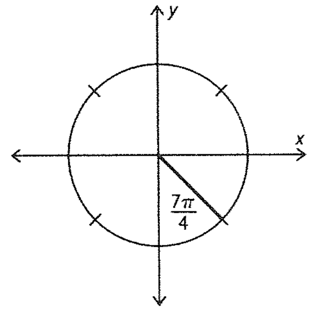
$\frac{5\pi}{6}$ is located in the second quadrant. The related angle is $\frac{\pi}{6}$, and secant is negative in the second quadrant.

e)



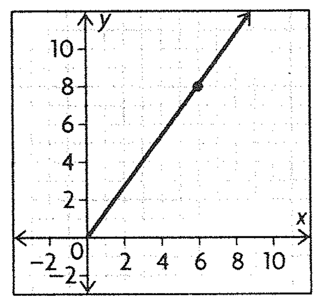
$\frac{2\pi}{3}$ is located in the second quadrant. The related angle is $\frac{\pi}{3}$, and cosine is negative in the second quadrant.

f)



$\frac{7\pi}{4}$ is located in the fourth quadrant. The related angle is $\frac{\pi}{4}$, and cotangent is negative in the fourth quadrant.

2. a) i)



ii) $r^2 = 6^2 + 8^2$

$r^2 = 36 + 64$

$r^2 = 100$

$r = 10$

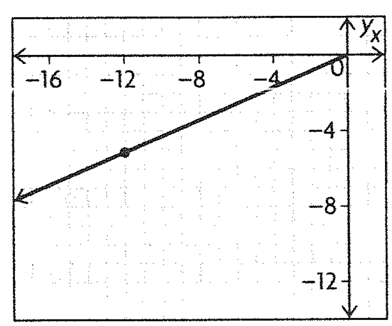
iii) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$,

$\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

iv) $\sin^{-1}\left(\frac{4}{5}\right) \doteq 0.93$

$\theta \doteq 0.93$

b) i)

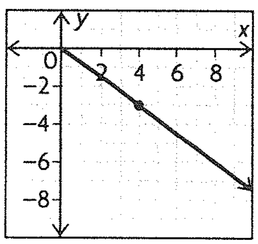


ii) $r^2 = (-12)^2 + (-5)^2$
$r^2 = 144 + 25$
$r^2 = 169$
$r = 13$

iii) $\sin\theta = -\frac{5}{13}, \cos\theta = -\frac{12}{13}, \tan\theta = \frac{5}{12},$
$\csc\theta = -\frac{13}{5}, \sec\theta = -\frac{13}{12}, \cot\theta = \frac{12}{5}$

iv) $\sin^{-1}\left(-\frac{5}{13}\right) \doteq -0.395$
$\theta \doteq \pi + 0.395 \doteq 3.54$

c) i)

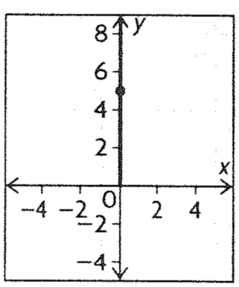


ii) $r^2 = 4^2 + (-3)^2$
$r^2 = 16 + 9$
$r^2 = 25$
$r = 5$

iii) $\sin\theta = -\frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = -\frac{3}{4},$
$\csc\theta = -\frac{5}{3}, \sec\theta = \frac{5}{4}, \cot\theta = -\frac{4}{3}$

iv) $\sin^{-1}\left(-\frac{3}{5}\right) \doteq -0.64$
$\theta \doteq 2\pi - 0.64 \doteq 5.64$

d) i)

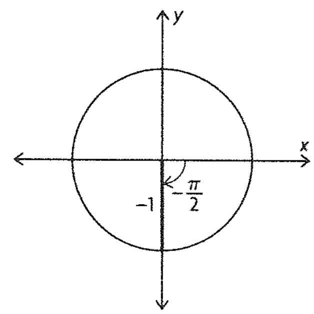


ii) $r^2 = 0^2 + 5^2$
$r^2 = 0 + 25$
$r^2 = 25$
$r = 5$

iii) $\sin\theta = \frac{5}{5} = 1, \cos\theta = \frac{0}{5} = 0,$
$\tan\theta = \frac{5}{0} = \text{undefined},$
$\csc\theta = \frac{5}{5} = 1, \sec\theta = \frac{5}{0} = \text{undefined},$
$\cot\theta = \frac{0}{5} = 0$

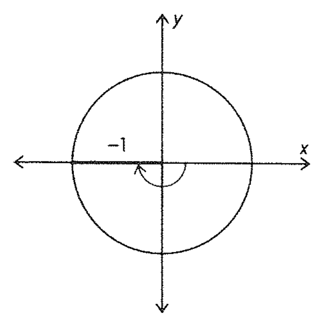
iv) $\sin^{-1}(1) \doteq 1.57$
$\theta \doteq \frac{\pi}{2}$

3. a)



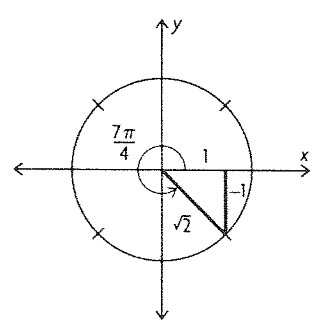
$x = 0, y = -1, r = 1$
$\sin\left(-\frac{\pi}{2}\right) = -1, \cos\left(-\frac{\pi}{2}\right) = 0,$
$\tan\left(-\frac{\pi}{2}\right) = \text{undefined}, \csc\left(-\frac{\pi}{2}\right) = -1,$
$\sec\left(-\frac{\pi}{2}\right) = \text{undefined}, \cot\left(-\frac{\pi}{2}\right) = 0$

b)



$x = -1, y = 0, r = 1$
$\sin(-\pi) = 0, \cos(-\pi) = -1, \tan(-\pi) = 0,$
$\csc(-\pi) = \text{undefined}, \sec(-\pi) = -1,$
$\cot(-\pi) = \text{undefined}$

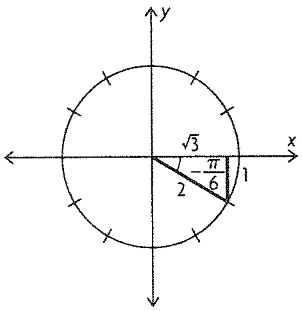
c)



$x = 1, y = -1, r = \sqrt{2}$

$$\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2},$$
$$\tan\left(\frac{7\pi}{4}\right) = -1, \csc\left(\frac{7\pi}{4}\right) = -\sqrt{2},$$
$$\sec\left(\frac{7\pi}{4}\right) = \sqrt{2}, \cot\left(\frac{7\pi}{4}\right) = -1$$

d)



$$x = \sqrt{3}, y = -1, r = 2$$
$$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}, \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},$$
$$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}, \csc\left(-\frac{\pi}{6}\right) = -2,$$
$$\sec\left(-\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{3}, \cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$$

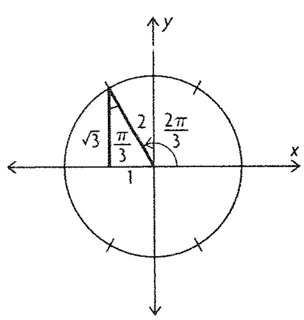
4. a) This is in the second quadrant where sine is positive. Sine is also positive in the first quadrant. So, an equivalent expression would be $\sin \frac{\pi}{6}$.

b) This is in the fourth quadrant where cosine is positive. Cosine is also positive in the first quadrant. So, an equivalent expression would be $\cos \frac{\pi}{3}$.

c) This is in the fourth quadrant where cotangent is negative. Cotangent is also negative in the second quadrant. So, an equivalent expression would be $\cot \frac{3\pi}{4}$.

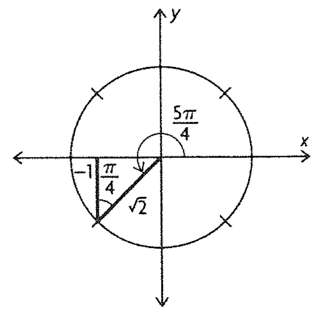
d) This is in the third quadrant where secant is negative. Secant is also negative in the second quadrant. So, an equivalent expression would be $\sec \frac{5\pi}{6}$.

5. a)



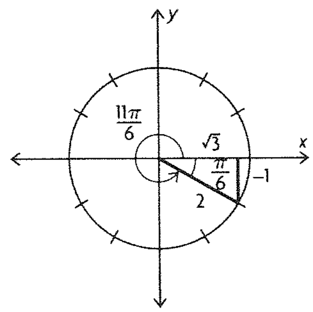
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

b)



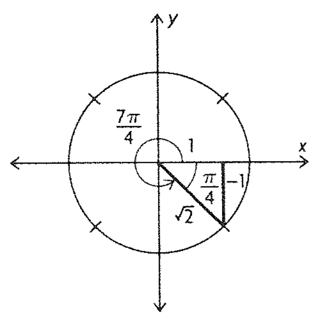
$$\cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

c)



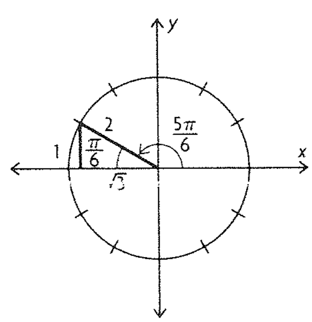
$$\tan\left(\frac{11\pi}{6}\right) = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

d)



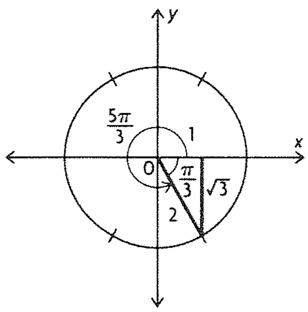
$$\sin\left(\frac{7\pi}{4}\right) = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

e)



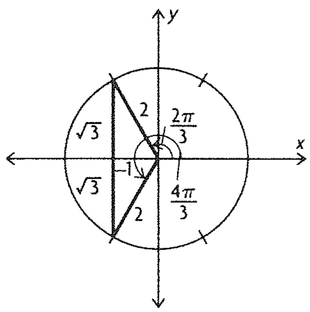
$$\csc\left(\frac{5\pi}{6}\right) = \frac{2}{1} = 2$$

f)



$$\sec\left(\frac{5\pi}{3}\right) = \frac{2}{1} = 2$$

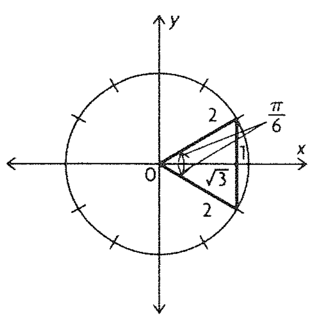
6. a)



$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cancel{\frac{2\pi}{3}}, \frac{4\pi}{3}$$

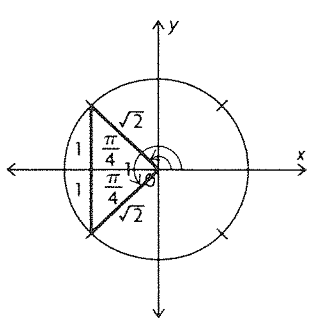
b)



$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cancel{\frac{\pi}{6}}, \frac{11\pi}{6}$$

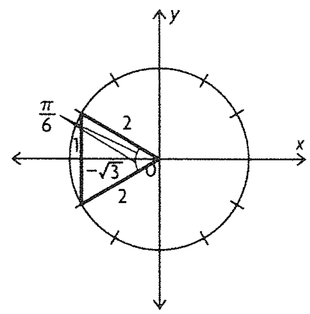
c) $-\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$



$$\cos\theta = -\frac{1}{\sqrt{2}}$$

$$\theta = \cancel{\frac{3\pi}{4}}, \frac{5\pi}{4}$$

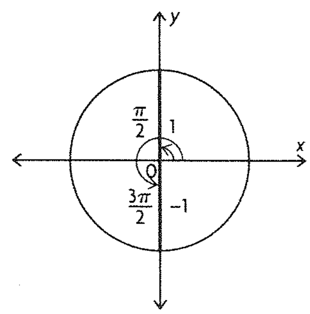
d)



$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \cancel{\frac{5\pi}{6}}, \frac{7\pi}{6}$$

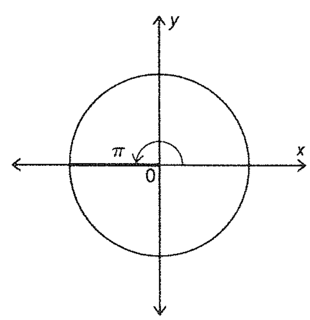
e)



$$\cos\theta = 0$$

$$\theta = \cancel{\frac{\pi}{2}}, \frac{3\pi}{2}$$

f)



$$\cos\theta = -1$$

$$\theta = \pi$$

7. a) $(-7, 8)$ is in the second quadrant.

$$\tan^{-1}\left(\frac{8}{-7}\right) \doteq -0.852$$

$$\theta \doteq \pi - 0.852 \doteq 2.29$$

b) $(12, 2)$ is in the first quadrant.

$$\tan^{-1}\left(\frac{2}{12}\right) \doteq 0.17$$

$$\theta \doteq 0.17$$

c) (3, 11) is in the first quadrant.

$\tan^{-1}\left(\frac{11}{3}\right) \doteq 1.30$

$\theta \doteq 1.30$

d) $(-4, -2)$ is in the third quadrant.

$\tan^{-1}\left(\frac{-2}{-4}\right) \doteq 0.464$

$\theta \doteq \pi + 0.464 \doteq 3.61$

e) (9, 10) is in the first quadrant.

$\tan^{-1}\left(\frac{10}{9}\right) \doteq 0.84$

$\theta \doteq 0.84$

f) $(6, -1)$ is in the fourth quadrant.

$\tan^{-1}\left(\frac{-1}{6}\right) \doteq -0.165$

$\theta \doteq 2\pi - 0.165 \doteq 6.12$

8. a) This is in the second quadrant where cosine is negative. Cosine is also negative in the third quadrant. So, an equivalent expression would be $\cos\frac{5\pi}{4}$.

b) This is in the fourth quadrant where tangent is negative. Tangent is also negative in the second quadrant. So, an equivalent expression would be $\tan\frac{5\pi}{6}$.

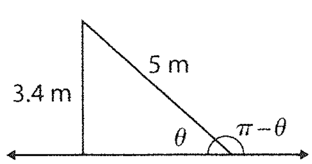
c) This is in the fourth quadrant where cosecant is negative. Cosecant is also negative in the third quadrant. So, an equivalent expression would be $\csc\frac{4\pi}{3}$.

d) This is in the second quadrant where cotangent is negative. Cotangent is also negative in the fourth quadrant. So, an equivalent expression would be $\cot\frac{5\pi}{3}$.

e) This is in the fourth quadrant where sine is negative. Sine is also negative in the third quadrant. So, an equivalent expression would be $\sin\frac{7\pi}{6}$.

f) This is in the fourth quadrant where secant is positive. Secant is also positive in the first quadrant. So, an equivalent expression would be $\sec\frac{\pi}{4}$.

9.

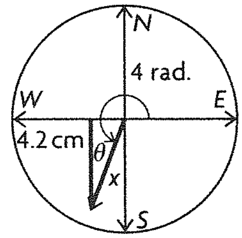


$\sin\theta = \frac{3.4}{5}$

$\theta = \sin^{-1}\left(\frac{3.4}{5}\right) \doteq 0.748$

$\pi - 0.748 \doteq 2.39$

10.

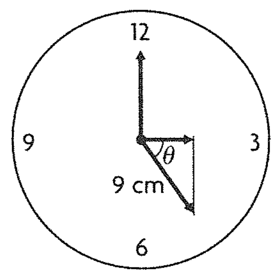


$\theta = 4 - \pi \doteq 0.8584$

$\sin(0.8584) = \frac{4.2}{x}$

$x = \frac{4.2}{\sin(0.8584)} \doteq 5.55 \text{ cm}$

11.



Use a proportion to find θ.

$\frac{10 \text{ sec}}{60 \text{ sec}} = \frac{r \text{ rad}}{2\pi \text{ rad}}$

$r \doteq 1.05 \text{ rad}$

$\cos(1.05) = \frac{x}{9}$

$x = 9\cos(1.05)$

$x \doteq 4.5 \text{ cm}$

12. Draw the angle and determine the measure of the reference angle. Use the CAST rule to determine the sign of each of the ratios in the quadrant in which the angle terminates. Use this sign and the value of the ratios of the reference angle to determine the values of the primary trigonometric ratios for the given angle.

13. a) It lies in the second or third quadrant because cosine is negative in these quadrants.

b) $x = -5$, $r = 13$, $y = ?$

$13^2 - (-5)^2 = y^2$

$169 - 25 = y^2$

$144 = y^2$

$12 = y$

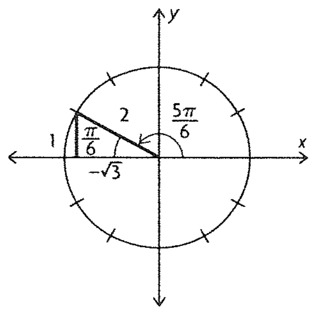
$$\sin\theta = \frac{12}{13} \text{ or } -\frac{12}{13},$$
$$\tan\theta = \frac{12}{5} \text{ or } -\frac{12}{5},$$
$$\sec\theta = -\frac{13}{5},$$
$$\csc\theta = \frac{13}{12} \text{ or } -\frac{13}{12},$$
$$\cot\theta = \frac{5}{12} \text{ or } -\frac{5}{12}$$

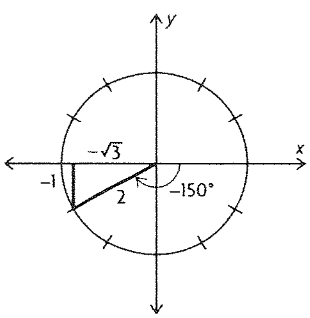
c) $\sin\theta = \dfrac{12}{13}$

$$\theta = \sin^{-1}\left(\frac{12}{13}\right)$$
$$\theta \doteq 1.176$$

In the second quadrant, $\pi - 1.176 \doteq 1.97$.
In the second quadrant, $\pi + 1.176 \doteq 4.32$.

14.





By examining the special triangles, we see
$$\cos\left(\frac{5\pi}{6}\right) = \cos(-150^\circ) = -\frac{\sqrt{3}}{2}$$

15. $2\left(\sin^2\left(\dfrac{11\pi}{6}\right)\right) - 1$

$$= 2\left(-\frac{1}{2}\right)^2 - 1$$
$$= 2\left(\frac{1}{4}\right) - 1$$
$$= -\frac{1}{2}$$

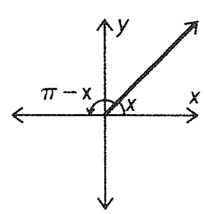
$$\left(\sin^2\frac{11\pi}{6}\right) - \left(\cos^2\frac{11\pi}{6}\right)$$
$$= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} - \frac{3}{4}$$
$$= -\frac{1}{2}$$

$$2\left(\sin^2\left(\frac{11\pi}{6}\right)\right) - 1 = \left(\sin^2\frac{11\pi}{6}\right) - \left(\cos^2\frac{11\pi}{6}\right)$$

16. $\sin\left(\dfrac{\pi}{6}\right) = \dfrac{8}{AB}$

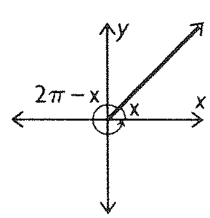
$$AB = \frac{8}{\sin\left(\frac{\pi}{6}\right)}$$
$$AB = 16$$
$$(AD)^2 = 8^2 + 8^2$$
$$(AD)^2 = 64 + 64$$
$$(AD)^2 = 128$$
$$(AD) = \sqrt{128} = 8\sqrt{2}$$
$$\sin D = \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2};$$
$$\cos D = \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2};$$
$$\tan D = \frac{8}{8} = 1$$

17. a)



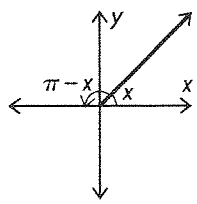
The first and second quadrants both have a positive y-value.

b)



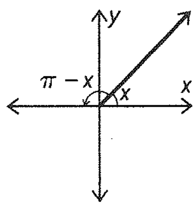
The first quadrant has a positive y-value, and the fourth quadrant has a negative y-value.

c)



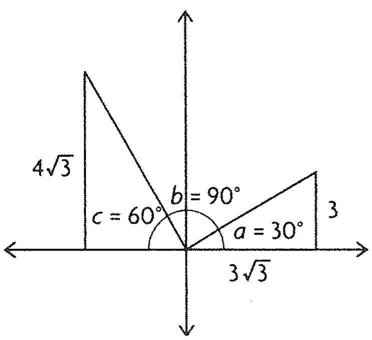
The first quadrant has a positive x-value, and the second quadrant has a negative x-value.

d)



The first quadrant has a positive x-value and a positive y-value, and the third quadrant has a negative x-value and a negative y-value.

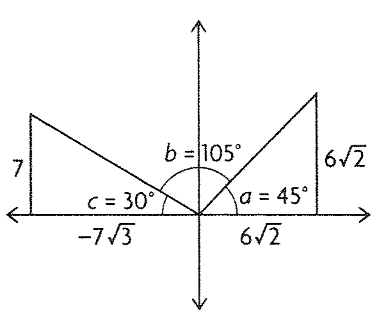
18.



$$\tan a = \frac{3}{3\sqrt{3}}$$
$$a = \tan^{-1}\left(\frac{3}{3\sqrt{3}}\right)$$
$$a = 30°$$
$$\tan c = \frac{4\sqrt{3}}{4}$$
$$c = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$$
$$c = 60°$$
$$b = 180° - 30° - 60° = 90°$$

$\sin 90° = 1$

19.

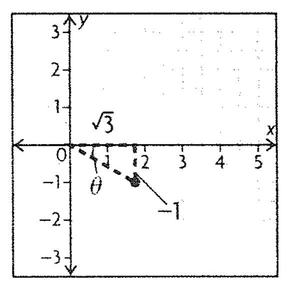


$$\tan a = \frac{6\sqrt{2}}{6\sqrt{2}}$$
$$a = \tan^{-1}\left(\frac{1}{1}\right)$$
$$a = 45°$$
$$\tan c = \frac{7}{7\sqrt{3}}$$
$$c = \tan^{-1}\left(\frac{7}{7\sqrt{3}}\right)$$
$$c = 30°$$
$$b = 180° - 30° - 45° = 105°$$

$\cos 150° \doteq -0.26$

20. The ranges of the cosecant and secant functions are both $\{y \in \mathbf{R} \mid -1 \geq y \text{ or } y \geq 1\}$. In other words, the values of these functions can never be between -1 and 1. For the values of these functions to be between -1 and 1, the values of the sine and cosine functions would have to be greater than 1 and less than -1, which is never the case.

21. The terminal arm is in the fourth quadrant. Cotangent is the ratio of adjacent side to opposite side. The given information leads to the figure shown below.



This is a special triangle, and the hypotenuse is 2.

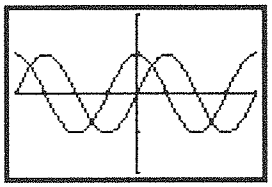
$$\sin\theta\cot\theta - \cos^2\theta = -\frac{1}{2}\left(-\sqrt{3}\right) - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{\sqrt{3}}{2} - \frac{3}{4}$$
$$= \frac{2\sqrt{3} - 3}{4}$$

6.3 Exploring Graphs of the Primary Trigonometric Functions, p. 336

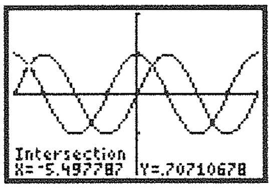
1. a) $y = \sin\theta$ and $y = \cos\theta$ have the same period, axis, amplitude, maximum value, minimum value, domain, and range. They have different y- and θ-intercepts.

b) $y = \sin\theta$ and $y = \tan\theta$ have no characteristics in common except for their y-intercept and zeros.

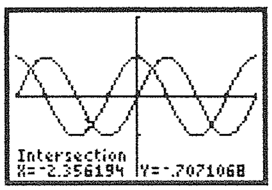
2. a)



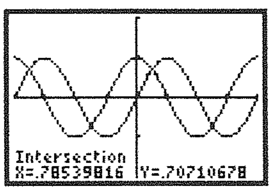
b)



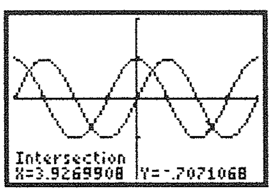
$\theta = -5.50$



$\theta = -2.36$



$\theta = 0.79$



$\theta = 3.93$

c) i) The graph of $y = \sin\theta$ intersects the θ-axis at $0, \pm\pi, \pm 2\pi, \ldots$

$t_n = n\pi, n \in \mathbf{I}$

ii) The maximum value occurs at $\frac{\pi}{2}$ and every 2π, since the period is 2π.

$t_n = \frac{\pi}{2} + 2n\pi, n \in \mathbf{I}$

iii) The minimum value occurs at $\frac{3\pi}{2}$ and every 2π, since the period is 2π.

$t_n = \frac{3\pi}{2} + 2n\pi, n \in \mathbf{I}$

3. a) The graph of $y = \cos\theta$ intersects the θ-axis at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

$t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

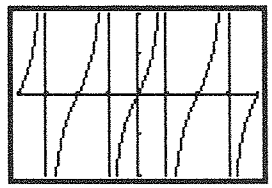
b) The maximum values occur at 0 and every 2π, since the period is 2π.

$t_n = 2n\pi, n \in \mathbf{I}$

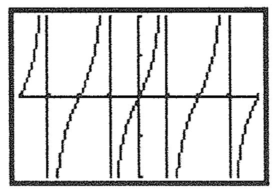
c) The minimum value occurs at π and every 2π, since the period is 2π.

$t_n = -\pi + 2n\pi, n \in \mathbf{I}$

4. Here is the graph of $y = \frac{\sin x}{\cos x}$:



Here is the graph of $y = \tan x$:



The two graphs appear to be identical.

5. a) The graph of $y = \tan\theta$ intersects the θ-axis at $0, \pm\pi, \pm 2\pi, \ldots$

$t_n = n\pi, n \in \mathbf{I}$

b) The graph of $y = \tan\theta$ has vertical asymptotes at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

$t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

6.4 Transformations of Trigonometric Functions, pp. 343–346

1. a) period: $\frac{2\pi}{|k|} = \frac{2\pi}{|4|} = \frac{\pi}{2}$

amplitude: $|a| = |0.5| = 0.5$

horizontal translation: $d = 0$

equation of the axis: $y = 0$

b) period: $\frac{2\pi}{|k|} = \frac{2\pi}{|1|} = 2\pi$

amplitude: $|a| = |1| = 1$

horizontal translation: $d = \frac{\pi}{4}$

equation of the axis: $y = 3$

c) period: $\frac{2\pi}{|k|} = \frac{2\pi}{3}$

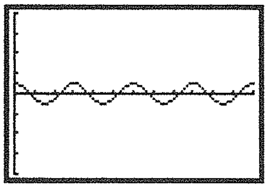
amplitude: $|a| = |2| = 2$

horizontal translation: $d = 0$

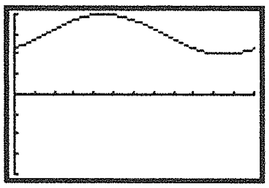
equation of the axis: $y = -1$

d) period: $\frac{2\pi}{|k|} = \frac{2\pi}{|-2|} = \pi$
amplitude: $|a| = |5| = 5$
horizontal translation: $d = \frac{\pi}{6}$
equation of the axis: $y = -2$

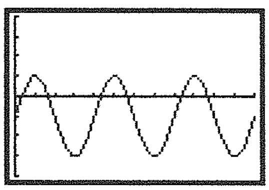
2. For $y = 0.5\cos(4x)$



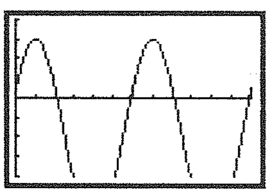
For $y = \sin\left(x - \frac{\pi}{4}\right) + 3$



For $y = 2\sin(3x) - 1$

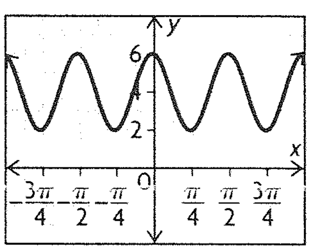


For $y = 5\cos\left(-2x + \frac{\pi}{3}\right) - 2$



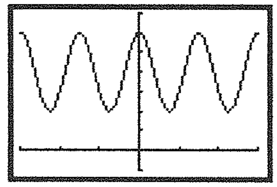
Only the last one is cut off.

3.



$y = -2\cos\left(4\left(x + \frac{\pi}{4}\right)\right) + 4$

period: $\frac{2\pi}{|k|} = \frac{2\pi}{|4|} = \frac{\pi}{2}$
amplitude: $|a| = |-2| = 2$
horizontal translation: $d = -\frac{\pi}{4}$ units to the left
equation of the axis: $y = 4$



4. $y = a\sin(k(x - d)) + c$

a) $a = 25$

period: $\frac{2\pi}{|k|} = \pi$
$k = 2$
$f(x) = 25\sin(2x) - 4$

b) $a = \frac{2}{5}$

period: $\frac{2\pi}{|k|} = 10$
$k = \frac{\pi}{5}$
$f(x) = \frac{2}{5}\sin\left(\frac{\pi}{5}x\right) + \frac{1}{15}$

c) $a = 80$

period: $\frac{2\pi}{|k|} = 6\pi$
$k = \frac{1}{3}$
$f(x) = 80\sin\left(\frac{1}{3}x\right) - \frac{9}{10}$

d) $a = 11$

period: $\frac{2\pi}{|k|} = \frac{1}{2}$
$k = 4\pi$
$f(x) = 11\sin(4\pi x)$

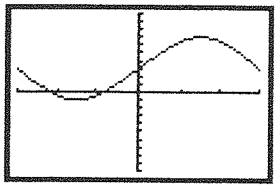
5. a) period $= 2\pi$, amplitude $= 18$, equation of the axis is $y = 0$;
$y = 18\sin x$

b) period $= 4\pi$, amplitude $= 6$, equation of the axis is $y = -2$;
$y = +6\sin(0.5x) - 2$

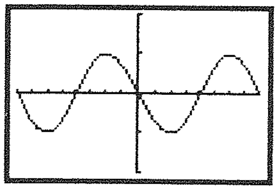
c) period $= 6\pi$, amplitude $= 2.5$, equation of the axis is $y = 6.5$;
$y = -2.5\cos\left(\frac{1}{3}x\right) + 6.5$

d) period $= 4\pi$, amplitude $= 2$, equation of the axis is $y = -1$;
$y = -2\cos\left(\frac{1}{2}x\right) - 1$

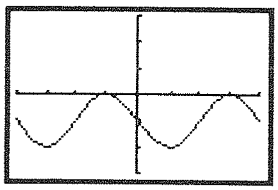
6. a) vertical stretch by a factor of 4, vertical translation 3 units up



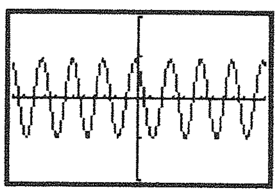
b) reflection in the x-axis, horizontal stretch by a factor of 4



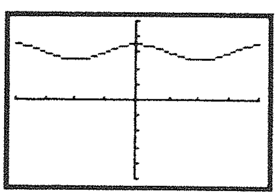
c) horizontal translation π to the right, vertical translation 1 unit down



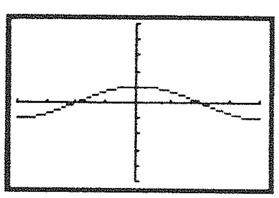
d) horizontal compression by a factor of $\frac{1}{4}$, horizontal translation $\frac{\pi}{6}$ to the left



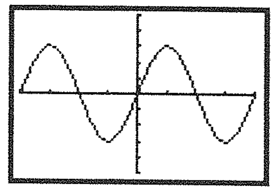
7. a) $f(x) = \frac{1}{2}\cos x + 3$



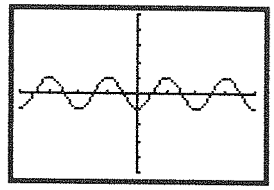
b) $f(x) = \cos\left(-\frac{1}{2}x\right)$



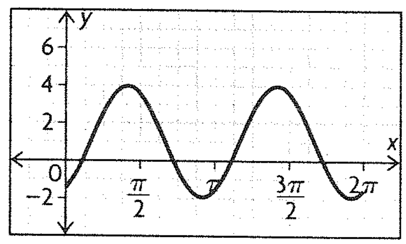
c) $f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$

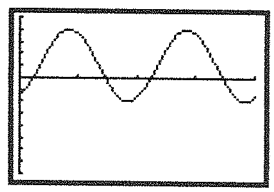


d) $f(x) = \cos\left(2\left(x + \frac{\pi}{2}\right)\right)$

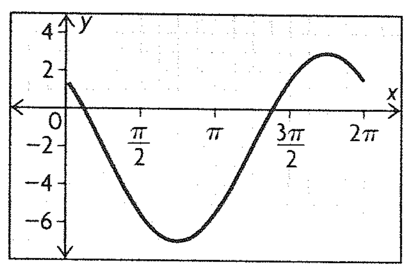


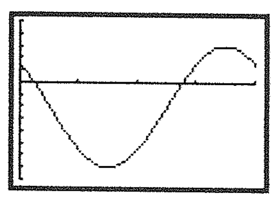
8. a)



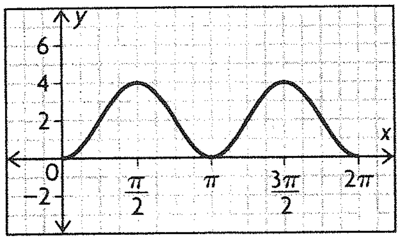


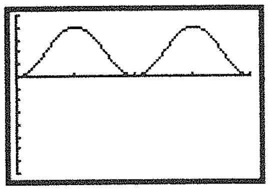
b)



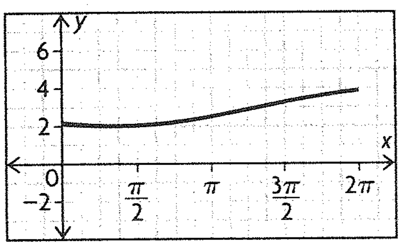


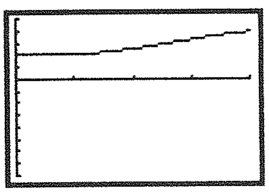
c)



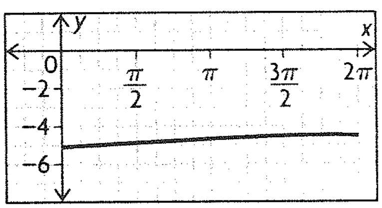


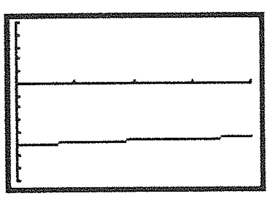
d)



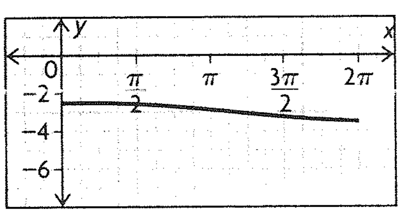


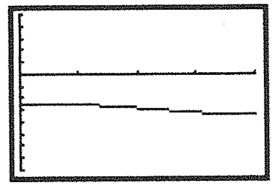
e)





f)





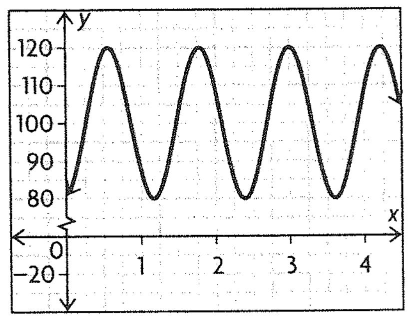
9. a) period: $\frac{2\pi}{|k|} = \frac{5\pi}{3}$

$$k = \frac{6}{5}$$

The period of the function is $\frac{6}{5}$.
This represents the time between one beat of a person's heart and the next beat.

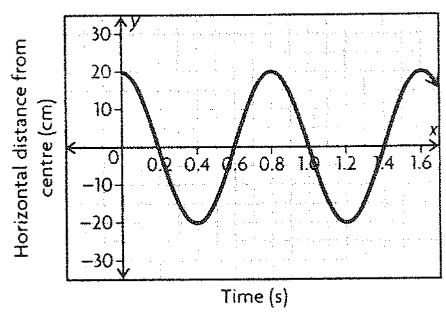
b) $P(60) = -20\cos\left(\frac{5\pi}{3}(60)\right) + 100 = 80$

c)



d) The range for the function is between 80 and 120. The range means the lowest blood pressure is 80 and the highest blood pressure is 120.

10. a)



b) There is a vertical stretch by a factor of 20. The period is 0.8 s.

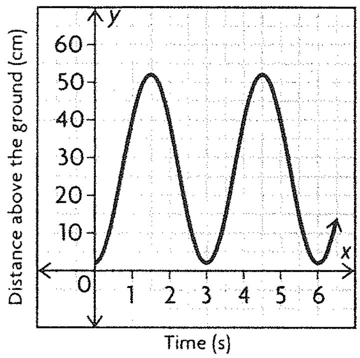
$$\frac{2\pi}{k} = 0.8$$

$$k = \frac{5\pi}{2}$$

There is a horizontal compression by a factor of $\frac{1}{|k|} = \frac{2}{5\pi}$.
There is a horizontal translation 0.2 to the left.

c) $y = 20\sin\left(\frac{5\pi}{2}(x + 0.2)\right)$

11. a)



b) vertical stretch by a factor of 25, reflection in the x-axis, vertical translation 27 units up; the period is 3 s.

$$\frac{2\pi}{k} = 3$$

$$k = \frac{2\pi}{3}$$

horizontal compression by a factor of $\frac{1}{|k|} = \frac{3}{2\pi}$

c) $y = -25\cos\left(\frac{2\pi}{3}x\right) + 27$

12. By looking at the difference in the x-values of the two maximums, $-\frac{5\pi}{7}$ and $-\frac{3\pi}{7}$, we see that the period is $\frac{2\pi}{7}$.

13. Answers may vary. For example, $\left(\frac{14\pi}{13}, 5\right)$. Since the maximum is 4 units above $y = 9$, the minimum would be at $y = 5$. If the period of the function is 2π, then the minimum would be at $\frac{\pi}{13} + \pi$ of $\frac{14\pi}{13}$.

14. a) This is a cosine function with amplitude = 1.

$$\text{period} = \frac{2\pi}{0.5} = 4\pi$$

$$y = \cos(4\pi x)$$

b) This is a sine function with a reflection in the x-axis and an amplitude = 2.

$$\text{period} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$y = -2\sin\left(\frac{\pi}{4}x\right)$$

c) The y-axis is $y = -1$ and the amplitude is 4. The function is shifted horizontally to the right by 10.

$$\text{period} = \frac{2\pi}{40} = \frac{\pi}{20}$$

$$y = 4\sin\left(\frac{\pi}{20}(x - 10)\right) - 1$$

15.

Start with graph of $y = \sin x$.

↓

Reflect in the x-axis and stretch vertically by a factor of 2 to produce graph of $y = -2\sin x$.

↓

Stretch horizontally by a factor of 2 to produce graph of $y = -2\sin(0.5x)$.

↓

Translate $\frac{\pi}{4}$ units to the right to produce graph of $y = -2\sin\left(0.5\left(x - \frac{\pi}{4}\right)\right)$.

↓

Translate 3 units up to produce graph of $y = -2\sin\left(0.5\left(x - \frac{\pi}{4}\right)\right) + 3$.

16. a) The car starts at the closest distance to the pole which is 100 m.

b) The centre of the track is 400 m from the pole because it is half the distance between the closest and furthest point.

c) The radius is 400 − 100 = 300 m.

d) The period of the function is 80 s. This is how long it takes to complete one lap.

e) $\frac{2\pi(300)}{80}$ m/s $\doteq$ 23.56194 m/s

Mid-Chapter Review, p. 349

1. a) $\frac{\cancel{\pi}}{8}\ \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi\ \text{radians}}}\right) = 22.5°$

b) $4\cancel{\pi\ \text{radians}} \times \left(\frac{180°}{\cancel{\pi\ \text{radians}}}\right) = 720°$

c) $5\ \cancel{\text{radians}} \times \left(\frac{180°}{\pi\ \cancel{\text{radians}}}\right) \doteq 286.5°$

d) $\frac{11\cancel{\pi}}{12}\ \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi\ \text{radians}}}\right) = 165°$

2. a) $125° = 125° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) \doteq 2.2 \text{ radians}$

b) $450° = 450° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) \doteq 7.9 \text{ radians}$

c) $5° = 5° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) \doteq 0.1 \text{ radians}$

d) $330° = 330° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) \doteq 5.8 \text{ radians}$

e) $215° = 215° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) \doteq 3.8 \text{ radians}$

f) $-140° = -140° \times \left(\dfrac{\pi \text{ radians}}{180°}\right) \doteq -2.4 \text{ radians}$

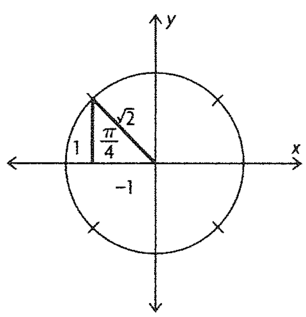
3. a) $10(2\pi) = 20\pi$

b) $\omega = \dfrac{20\pi}{5} = 4\pi \text{ radians/s}$

c) Circumference $= 2\pi(19) = 38\pi$

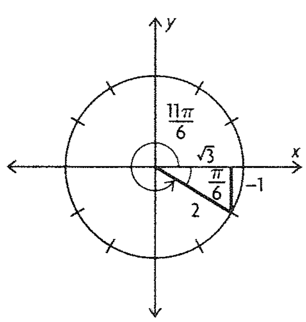
$38\pi \times 10 \text{ revolutions} = 380\pi \text{ cm}$

4. a)



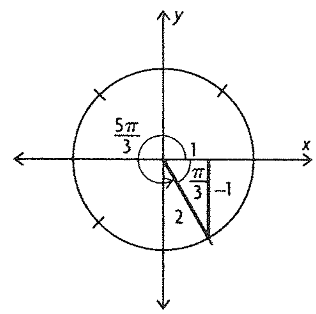
$\sin \dfrac{3\pi}{4} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

b)



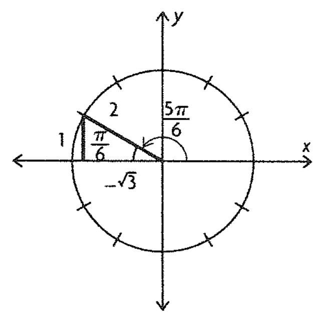
$\sin \dfrac{11\pi}{6} = -\dfrac{1}{2}$

c)



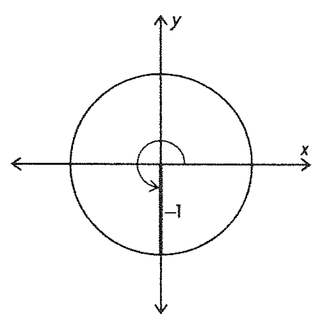
$\tan \dfrac{5\pi}{3} = -\sqrt{3}$

d)



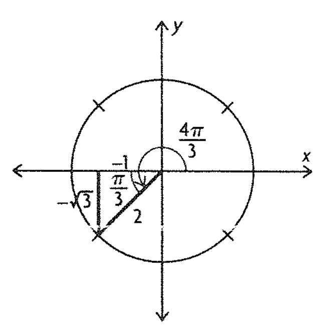
$\tan \dfrac{5\pi}{6} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

e)



$\cos \dfrac{3\pi}{2} = \dfrac{0}{1} = 0$

f)



$\cos \dfrac{4\pi}{3} = -\dfrac{1}{2}$

5. a) $(-3, 14)$ is in the second quadrant.

$\tan^{-1}\left(\frac{14}{-3}\right) \doteq -1.360$

$\theta \doteq \pi - 1.360 \doteq 1.78$

b) $(6, 7)$ is in the first quadrant.

$\tan^{-1}\left(\frac{7}{6}\right) \doteq 0.86$

c) $(1, 9)$ is in the first quadrant.

$\tan^{-1}\left(\frac{9}{1}\right) \doteq 1.46$

d) $(-5, -18)$ is in the third quadrant.

$\tan^{-1}\left(\frac{-18}{-5}\right) \doteq 1.30$

$\theta \doteq \pi + 1.30 \doteq 4.44$

e) $(2, 3)$ is in the first quadrant.

$\tan^{-1}\left(\frac{3}{2}\right) \doteq 0.98$

f) $(4, -20)$ is in the fourth quadrant.

$\tan^{-1}\left(\frac{-20}{4}\right) \doteq -1.373$

$\theta \doteq 2\pi - 1.360 \doteq 4.91$

6. a) This is in the second quadrant where sine is positive. Sine is also positive in the first quadrant. So, an equivalent expression would be $\sin \frac{\pi}{6}$.

b) This is in the fourth quadrant where cotangent is negative. Cotangent is also negative in the second quadrant. So, an equivalent expression would be $\cot \frac{3\pi}{4}$.

c) Secant is undefined at $-\frac{\pi}{2}$. It is also undefined at $\frac{\pi}{2}$. So, an equivalent expression would be $\sec \frac{\pi}{2}$.

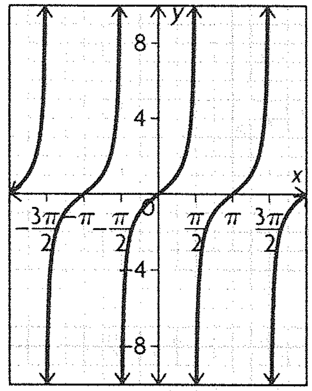
d) This is in the third quadrant where cosine is negative. Cosine is also negative in the second quadrant. So, an equivalent expression would be $\cos \frac{5\pi}{6}$.

7. a) $x = 0, \pm\pi, \pm 2\pi, \ldots; y = 0$

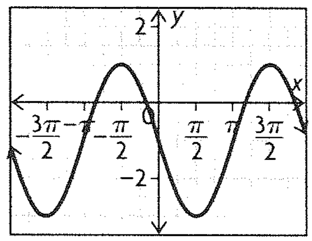
b) $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots; y = 1$

c) $x = 0, \pm\pi, \pm 2\pi, \ldots; y = 0$

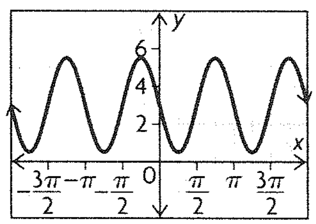
8. a)



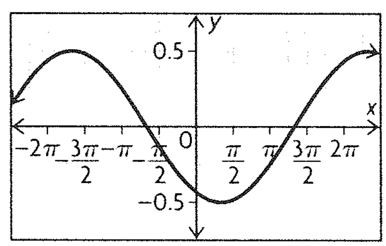
b)



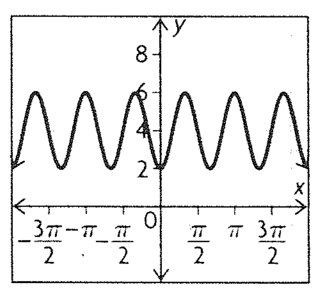
c)



d)



e)



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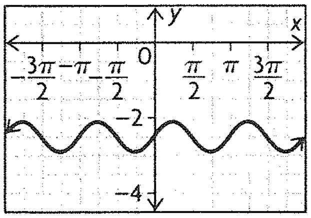


9. $y = \frac{1}{3}\sin\left(-3\left(x + \frac{\pi}{8}\right)\right) - 23$

6.5 Exploring Graphs of the Reciprocal Trigonometric Functions, p. 353

1. a) The graph of $y = \csc x$ has vertical asymptotes at $0, \pm\pi, \pm2\pi, \ldots$

$t_n = n\pi, n \in \mathbf{I}$

b) $y = \csc x$ has no maximum value.

c) $y = \csc x$ has no minimum value.

2. a) The graph of $y = \sec x$ has vertical asymptotes at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

$t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

b) $y = \sec x$ has no maximum value.

c) $y = \sec x$ has no minimum value.

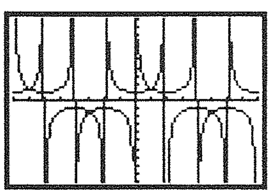
3. a) The graph of $y = \cot x$ has vertical asymptotes at $0, \pm\pi, \pm2\pi, \ldots$

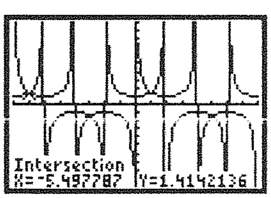
$t_n = n\pi, n \in \mathbf{I}$

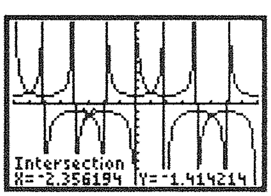
b) The graph of $y = \cot x$ intersects the x-axis at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

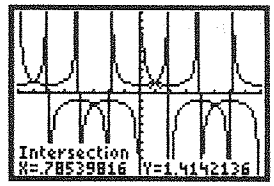
$t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

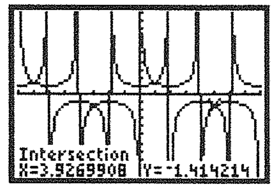
4.





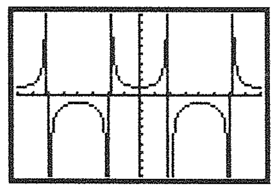






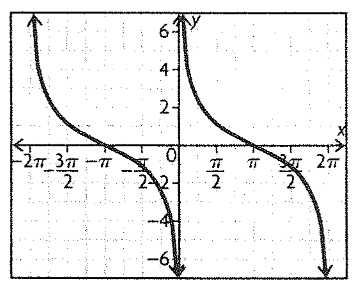
The values of x for which $y = \csc x$ and $y = \sec x$ intersect are $x = -5.50, -2.35, 0.79, 3.93$, the same values for which $y = \sin x$ and $y = \cos x$ were determined to intersect in Lesson 6.3.

5. Yes; the graphs of $y = \csc\left(x + \frac{\pi}{2}\right)$ and $y = \sec x$ are identical.

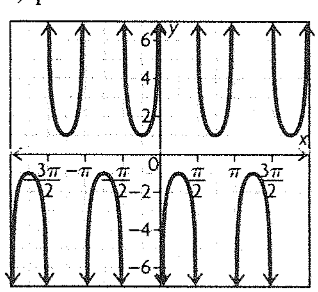


6. Answers may vary. For example, reflect the graph of $y = \tan x$ across the y-axis and then translate the graph $\frac{\pi}{2}$ units to the left.

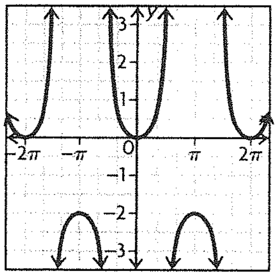
7. a) period $= 2\pi$



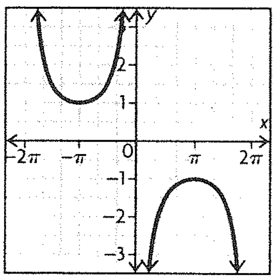
b) period $= \pi$



c) period $= 2\pi$



d) period $= 4\pi$



6.6 Modelling with Trigonometric Functions, p. 360–362

1. $y = 3\cos\left(\frac{2}{3}\left(x + \frac{\pi}{4}\right)\right) + 2$

2. For $x = \frac{\pi}{2}$,

$$y = 3\cos\left(\frac{2}{3}\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right) + 2$$

$$y = 3\cos\left(\frac{2}{3}\left(\frac{3\pi}{4}\right)\right) + 2$$

$$y = 3\cos\left(\frac{\pi}{2}\right) + 2$$

$$y = 0 + 2$$
$$y = 2$$

For $x = \frac{3\pi}{4}$

$$y = 3\cos\left(\frac{2}{3}\left(\frac{3\pi}{4} + \frac{\pi}{4}\right)\right) + 2$$

$$y = 3\cos\left(\frac{2}{3}(\pi)\right) + 2$$

$$y = 3(-0.5) + 2$$
$$y = -1.5 + 2$$
$$y = 0.5$$

For $x = \frac{11x}{6}$,

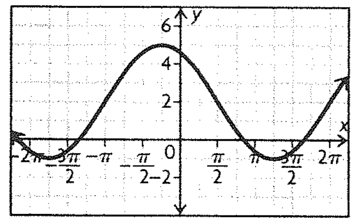
$$y = 3\cos\left(\frac{2}{3}\left(\frac{11\pi}{6} + \frac{\pi}{4}\right)\right) + 2$$

$$y = 3\cos\left(\frac{2}{3}\left(\frac{25\pi}{12}\right)\right) + 2$$

$$y = 3\cos\left(\frac{25\pi}{18}\right) + 2$$

$$y \doteq 0.973\ 94$$

3.



$x = 1.3$

4. amplitude and equation of the axis

5. a) The amplitude represents the radius of the circle in which the tip of the sparkler is moving.
b) The period represents the time it takes Mike to make one complete circle with the sparkler.
c) The equation of the axis represents the height above the ground of the centre of the circle in which the tip of the sparkler is moving.
d) A cosine function should be used because the starting point is at the highest point.

6. The amplitude of the function is 90 with the equation of the axis being $y = 30$.

$$k = \frac{2\pi}{24} = \frac{\pi}{12}$$

$$y = 90\sin\left(\frac{\pi}{12}x\right) + 30$$

7. The amplitude of the function is 250 with the equation of the axis being $y = 750$.
period $= 3$ seconds

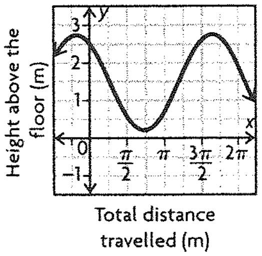
$$k = \frac{2\pi}{3}$$

$$y = 250\cos\left(\frac{2\pi}{3}x\right) + 750$$

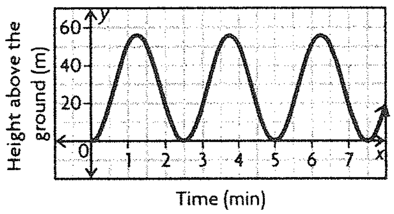
8. The amplitude of the function is 1.25 with the equation of the axis being $y = 1.5$. There is a reflection across the x-axis.
Circumference $= 2\pi(1.25) = 2.5\pi$

$$k = \frac{2\pi}{2.5\pi} = \frac{4}{5}$$

$$y = -1.25\sin\left(\frac{4}{5}x\right) + 1.5$$



9. 0.98 min $< t <$ 1.52 min,
3.48 min $< t <$ 4.02 min, 5.98 min $< t <$ 6.52 min



10. a) The amplitude of the function is $\frac{15.7 - 8.3}{2} = 3.7$ with the equation of the axis being $y = \frac{8.3 + 15.7}{2}$ or $y = 12$.

period = 365 days

$$k = \frac{2\pi}{365}$$

$$y = 3.7 \sin\left(\frac{2\pi}{365}(x - 172)\right) + 12$$

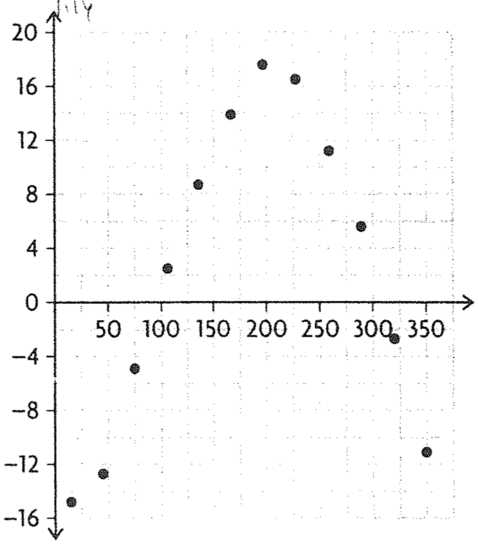
X

b) For $x = 30$,

$$y = 3.7 \sin\left(\frac{2\pi}{365}(30 - 142)\right) + 12$$

$y \doteq$ ~~13.87~~ 9.14 hours X

11.



The axis is at $\frac{-14.8 + 17.6}{2} = 1.4$. The amplitude is 16.2. The period is 365 days.

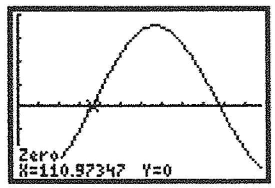
$$k = \frac{2\pi}{365}$$

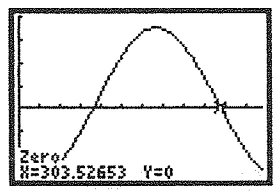
why sin

$$T(t) = 16.2 \sin\left(\frac{2\pi}{365}(t - 116)\right) + 1.4$$

X

Graph the equation on a graphing calculator to determine when the temperature is below 0 °C.





$0 < t < 111$ and $304 < t < 365$

12. The student should graph the height of the nail above the ground as a function of the total distance travelled by the nail, because the nail would not be travelling at a constant speed. If the student graphed the height of the nail above the ground as a function of time, the graph would not be sinusoidal.

13. The axis is at 3 m = 300 cm. The amplitudes of the minute hand and the second hand is 15 and of the hour hand is 8.

The period of the minute hand is 60.

$$k = \frac{2\pi}{60} = \frac{\pi}{30}$$

minute hand: $D(t) = 15 \cos\left(\frac{\pi}{30}t\right) + 300$;

The period of the second hand is 1.

$$k = \frac{2\pi}{1} = 2\pi$$

second hand: $D(t) = 15 \cos(2\pi t) + 300$;

The period of the hour hand is 720.

$$k = \frac{2\pi}{720} = \frac{\pi}{360}$$

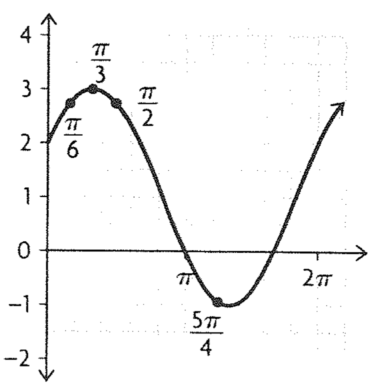
hour hand: $D(t) = 8 \cos\left(\frac{\pi}{360}t\right) + 300$

6.7 Rates of Change in Trigonometric Functions, pp. 369–373

1. a) The average rate of change is zero in the intervals of $0 < x < \pi$, $\pi < x < 2\pi$.
b) The average rate of change is negative in the intervals of $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $\frac{3\pi}{2} < x < \frac{5\pi}{2}$.
c) The average rate of change is positive in the intervals of $\frac{\pi}{2} < x < \frac{3\pi}{2}$, $\frac{5\pi}{2} < x < 3\pi$.
2. a) Two points where the instantaneous rate of change is zero are $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$.
b) Two points where the instantaneous rate of change is a negative value are $x = \frac{\pi}{2}$, $x = \frac{5\pi}{2}$.
c) Two points where the instantaneous rate of change is a positive value are $x = 0$, $x = 2\pi$.
3. Average rate of change for the interval $2 \le x \le 5$:

$$\left|\frac{0-0}{5-2}\right| = \left|\frac{0}{3}\right| = 0$$

4.



a) $0 \le x \le \frac{\pi}{2}$

$$\text{Average rate of change} = \frac{2.73 - 2}{\frac{\pi}{2} - 0} = \frac{0.73}{1.57} \doteq 0.465$$

b) $\frac{\pi}{6} \le x \le \frac{\pi}{2}$

$$\text{Average rate of change} = \frac{2.73 - 2.73}{\frac{\pi}{2} - \frac{\pi}{6}} = \frac{0}{1.047} = 0$$

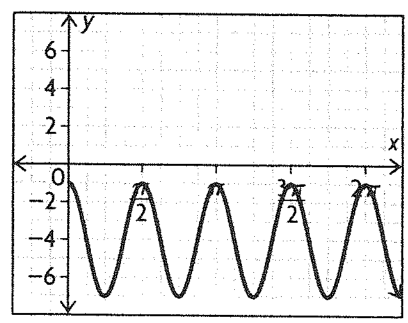
c) $\frac{\pi}{3} \le x \le \frac{\pi}{2}$

$$\text{Average rate of change} = \frac{2.73 - 3}{\frac{\pi}{2} - \frac{\pi}{3}} = \frac{-0.27}{0.5236} \doteq -0.5157$$

d) $\frac{\pi}{2} \le x \le \frac{5\pi}{4}$

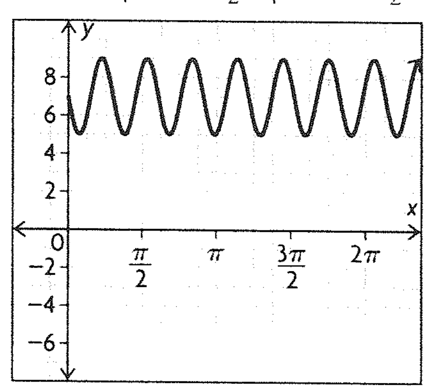
$$\text{Average rate of change} = \frac{-0.932 - 2.73}{\frac{5\pi}{4} - \frac{\pi}{2}} = \frac{-3.662}{2.356} \doteq -1.554$$

5.



a) The average rate of change is zero in the intervals of $0 < x < \frac{\pi}{2}$, $\pi < x < \frac{3\pi}{2}$.
b) The average rate of change is negative in the intervals of $0 < x < \frac{\pi}{4}$, $\pi < x < \frac{5\pi}{4}$.
c) The average rate of change is positive in the intervals of $\frac{\pi}{4} < x < \frac{\pi}{2}$, $\frac{5\pi}{4} < x < \frac{3\pi}{2}$.

6.



a) Two points where the instantaneous rate of change is zero are $x = \frac{1}{4}$, $x = \frac{3}{4}$.
b) Two points where the instantaneous rate of change is a negative value are $x = 0$, $x = 1$.
c) Two points where the instantaneous rate of change is a positive value are $x = \frac{1}{2}$, $x = \frac{3}{2}$.

7. a) $y = 6\cos(3x) + 2$ for $\frac{\pi}{4} \le x \le \pi$

For $x = \frac{\pi}{4}$,

$y = 6\cos\left(3\left(\frac{\pi}{4}\right)\right) + 2$

$y \doteq -2.2426$

For $x = \pi$,

$y = 6\cos(3(\pi)) + 2$

$y = -4$

$$\text{Average rate of change} = \frac{-2.2426 - (-4)}{\frac{\pi}{4} - \pi} = \frac{1.7574}{-2.3562} \doteq -0.7459$$

b) $y = -5\sin\left(\frac{1}{2}x\right) - 9$ for $\frac{\pi}{4} \le x \le \pi$

For $x = \frac{\pi}{4}$,

$y = -5\sin\left(\frac{1}{2}\left(\frac{\pi}{4}\right)\right) - 9$

$y \doteq -10.9134$

For $x = \pi$,

$y = -5\sin\left(\frac{1}{2}\pi\right) - 9$

$y = -14$

$$\text{Average rate of change} = \frac{-10.9134 - (-14)}{\frac{\pi}{4} - \pi} = \frac{3.0866}{-2.3562} \doteq -1.310$$

c) $y = \frac{1}{4}\cos(8x) + 6$ for $\frac{\pi}{4} \le x \le \pi$

For $x = \frac{\pi}{4}$,

$y = \frac{1}{4}\cos\left(8\left(\frac{\pi}{4}\right)\right) + 6$

$y = 6.25$

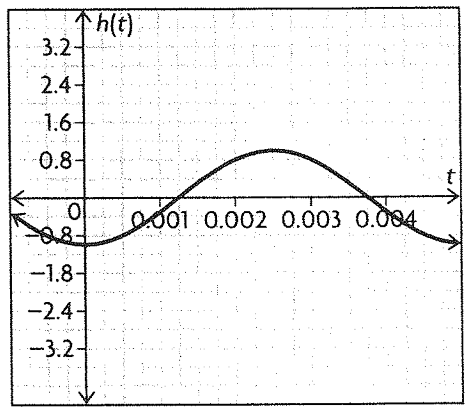
For $x = \pi$,

$y = \frac{1}{4}\cos(8\pi) + 6$

$y = 6.25$

$$\text{Average rate of change} = \frac{6.25 - 6.25}{\frac{\pi}{4} - \pi} = \frac{0}{-2.3562} = 0$$

8. The tip is at its minimum height at $t = 0$. Normally the sine function is 0 at 0, so the function in this case is translated to the right by $\frac{1}{4}$ of its period. The propeller makes 200 revolutions per second, so the period is $\frac{1}{200}$. The amplitude of the function is the length of the propeller, which is positive. Assume that it is 1 m for this exercise. Then the function that describes the height of the tip of the propeller is $h(t) = \sin\left(400\pi\left(t - \frac{1}{800}\right)\right)$. The graph of this function is shown below.



$t = \frac{1}{300} \doteq 0.0033$

From the graph, it is clear that the instantaneous rate of change at $t = \frac{1}{300}$ is negative.

9. a) The axis is at 20.2. So, the equation of the axis is $y = 20.2$ and the amplitude is 4.5.

$k = \frac{2\pi}{24} = \frac{\pi}{12}$

$R(t) = 4.5\cos\left(\frac{\pi}{12}t\right) + 20.2$

b) fastest: $t = 6$ months, $t = 18$ months, $t = 30$ months, $t = 42$ months;
slowest: $t = 0$ months, $t = 12$ months, $t = 24$ months, $t = 36$ months, $t = 48$ months

c) For $5 \le t \le 7$

$t = 5$

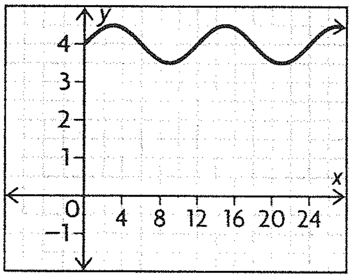
$R(5) = 4.5\cos\left(\frac{\pi}{12}(5)\right) + 20.2$

$R(5) \doteq 21.364$

$t = 7$

$$R(7) = 4.5\cos\left(\frac{\pi}{12}(7)\right) + 20.2$$
$R(7) \doteq 19.035$
Use the points (5, 21.364) and (7, 19.035).
$$\frac{19.035 - 21.364}{7 - 5} \doteq \frac{-2.329}{2} \doteq -1.164$$
$\doteq 1.164$ mice per owl/s

10. a)



The instantaneous rate of change appears to be at its greatest at 12 hours.

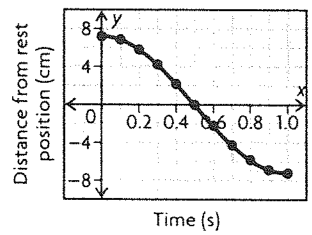
i) For $11 \le t \le 13$
$t = 11$
$$y = 0.5\sin\left(\frac{\pi}{6}(11)\right) + 4$$
$y = 3.75$
$t = 13$
$$y = 0.5\sin\left(\frac{\pi}{6}(13)\right) + 4$$
$y = 4.25$
Use the points (11, 3.75) and (13, 4.25).
$$\frac{4.25 - 3.75}{13 - 11} = \frac{0.5}{2} = 0.25 \text{ t/h}$$

ii) For $11.5 \le t \le 12.5$
$t = 11.5$
$$y = 0.5\sin\left(\frac{\pi}{6}(11.5)\right) + 4$$
$y \doteq 3.8706$
$t = 12.5$
$$y = 0.5\sin\left(\frac{\pi}{6}(12.5)\right) + 4$$
$y \doteq 4.1294$
Use the points (11.5, 3.8706) and (12.5, 4.1294).
$$\frac{4.1294 - 3.8706}{12.5 - 11.5} \doteq 0.2588 \text{ t/h}$$

iii) For $11.75 \le t \le 12.25$
$t = 11.75$
$$y = 0.5\sin\left(\frac{\pi}{6}(11.75)\right) + 4$$
$y \doteq 3.9347$
$t = 12.25$
$$y = 0.5\sin\left(\frac{\pi}{6}(12.25)\right) + 4$$
$y \doteq 4.0653$
Use the points (11.75, 3.9347 and (12.25, 4.0653).
$$\frac{4.0653 - 3.9347}{12.25 - 11.75} = \frac{0.1306}{0.5} = 0.2612 \text{ t/h}$$

b) The estimate calculated in part iii) is the most accurate. The smaller the interval, the more accurate the estimate.

11. a)



b) half of one cycle

c) $\dfrac{-7.2 - 7.2}{1 - 0} = -14.4$ cm/s

d) The bob is moving the fastest when it passes through its rest position. You can tell because the images of the balls are farthest apart at this point.

e) The pendulum's rest position is halfway between the maximum and minimum values on the graph. Therefore, at this point, the pendulum's instantaneous rate of change is at its maximum.

12. $h(t) = \sin\left(\frac{\pi}{5}t\right)$

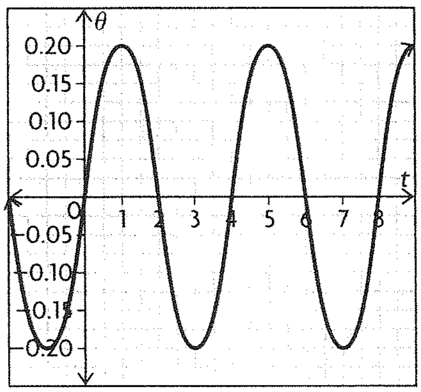
a) For $0 \le t \le 5$,
$$h(0) = \sin\left(\frac{\pi}{5}(0)\right) = 0$$
$$h(5) = \sin\left(\frac{\pi}{5}(5)\right) = 0$$
$$\frac{0 - 0}{5 - 0} = 0$$

b) For $5.5 \le t \le 6.5$
$t = 5.5$
$$h(5.5) = \sin\left(\frac{\pi}{5}(5.5)\right)$$
$h(5.5) = -0.309$
$$h(6.5) = \sin\left(\frac{\pi}{5}(6.5)\right)$$
$h(6.5) \doteq -0.809$

Use the points $(5.5, -0.309)$ and $(6.5, -0.809)$.

$$\frac{-0.809 - (-0.309)}{6.5 - 5.5} = \frac{-0.5}{1} = -0.5 \text{ m/s}$$

13. a)



b) When $t = 0, \theta = 0$. When $t = 1$, $\theta = 0.2$. The average rate of change is 0.2 radians/s.

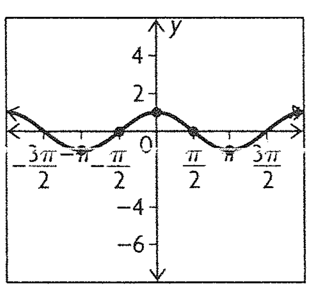
c) Answers may vary. For example, from the graph, it appears that the instantaneous rate of at $t = 1.5$ is about $-\frac{2}{3}$ radians/s.

d) The pendulum speed seems to be the greatest for $t = 0, 2, 4, 6$, and 8.

14. Answers may vary. For example, for $x = 0$, the instantaneous rate of change of $f(x) = \sin x$ is approximately 0.9003, while the instantaneous rate of change of $f(x) = 3 \sin x$ is approximately 2.7009. (The interval $-\frac{\pi}{4} < x < \frac{\pi}{4}$ was used.) Therefore, the instantaneous rate of change of $f(x) = 3 \sin x$ is at its maximum three times more than the instantaneous rate of change of $f(x) = \sin x$. However, there are points where the instantaneous rate of change is the same for the two functions. For example, at $x = \frac{\pi}{2}$, it is 0 for both functions.

15. a) By examining the graph of $f(x) = \sin x$, it appears that the instantaneous rate of change at the given values of x are $-1, 0, 1, 0$, and -1.

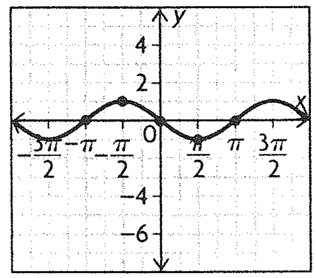
b)



The function is $f(x) = \cos x$. Based on this information, the derivative of $f(x) = \sin x$ is $\cos x$.

16. a) By examining the graph of $f(x) = \cos x$, it appears that the instantaneous rate of change at the given values of x are $0, 1, 0, -1$, and 0.

b)



The function is $f(x) = -\sin x$. Based on this information, the derivative of $f(x) = \cos x$ is $-\sin x$.

Chapter Review, pp. 376–377

1. Circumference $= 2\pi r = 2\pi(16) = 32\pi$

$$\frac{33}{32\pi} = \frac{x}{2\pi}$$

$$(32\pi)(x) = (33)(2\pi)$$

$$(32\pi)(x) = 66\pi$$

$$x = \frac{66\pi}{32\pi}$$

$$x = \frac{33}{16}$$

2. Circumference $= 2\pi r = 2\pi(75) = 150\pi$

$$\frac{x}{150\pi} = \frac{\frac{14\pi}{15}}{2\pi}$$

$$(2\pi)(x) = (150\pi)\left(\frac{14\pi}{15}\right)$$

$$(2\pi)(x) = 140\pi^2$$

$$x = 70\pi$$

3. a) $20° = 20° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{9}$ radians

b) $-50° = -50° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{-5\pi}{18}$ radians

c) $160° = 160° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{8\pi}{9}$ radians

d) $420° = 420° \times \left(\frac{\pi \text{ radians}}{180°}\right) = \frac{7\pi}{3}$ radians

4. a) $\frac{\pi}{4}$ radians;

$$\frac{\cancel{\pi}}{4} \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi} \cancel{\text{radians}}}\right) = 45°$$

b) $-\frac{5\pi}{4}$ radians;

$$-\frac{5\pi}{4}\cancel{\text{radians}} \times \left(\frac{180^\circ}{\pi\ \cancel{\text{radians}}}\right) = -225^\circ$$

c) $\frac{8\pi}{3}$ radians;

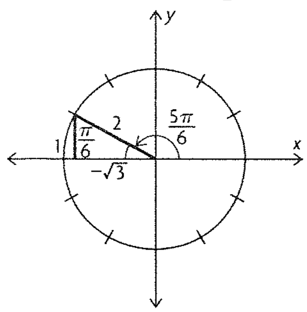
$$\frac{8\pi}{3}\cancel{\text{radians}} \times \left(\frac{180^\circ}{\pi\ \cancel{\text{radians}}}\right) = 480^\circ$$

d) $-\frac{2\pi}{3}$ radians;

$$-\frac{2\pi}{3}\cancel{\text{radians}} \times \left(\frac{180^\circ}{\pi\ \cancel{\text{radians}}}\right) = -120^\circ$$

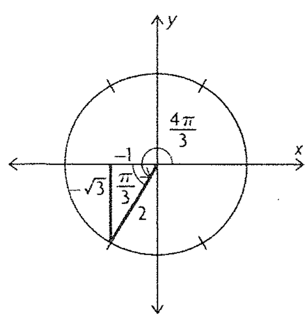
5. The functions must be located in the second or third quadrant, since $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$.

a)



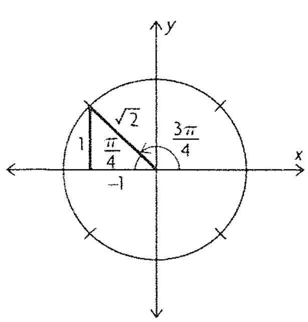
$$\sin^{-1}\frac{1}{2} = \frac{5\pi}{6}$$

b)



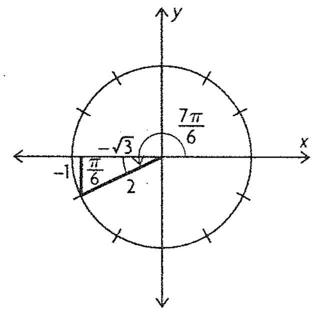
$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{4\pi}{3}$$

c) $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

d)



$$\sin^{-1}\left(-\frac{1}{2}\right) = \frac{7\pi}{6}$$

6. $\cos\theta = \frac{x}{r} = \frac{-5}{13}$

$$\begin{aligned} 13^2 &= (-5)^2 + y^2 \\ 169 - 25 &= y^2 \\ 144 &= y^2 \\ 12 &= y \end{aligned}$$

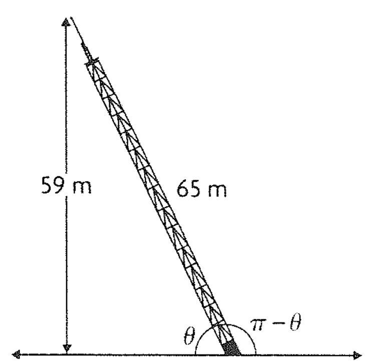
a) $\tan\theta = \frac{y}{x} = \frac{12}{13}$

b) $\sec\theta = \frac{r}{x} = -\frac{13}{5}$

c) Cosine is negative in the second and third quadrants.

$$\cos^{-1}\frac{-5}{13} \doteq 2.0 \text{ and } \pi + 2.0 \doteq 5.14$$

7.



$$\sin\theta = \frac{59}{65}$$

$$\theta = \sin^{-1}\frac{59}{65} = 1.14$$

$\pi - \theta = 2.00$

8. a) 2π radians

b) 2π radians

c) π radians

9. The axis is 2 and the amplitude is 5. It is shifted to the left $\frac{\pi}{3}$.

$$y = 5\sin\left(x + \frac{\pi}{3}\right) + 2$$

10. The axis is -1 and the amplitude is 3. It is shifted $\frac{3\pi}{4}$ units to the left. ~~It is reflected in the~~ x-axis. The period is π.

$$\cancel{k = \frac{2\pi}{\pi} = 2}$$

$$y = +3\cos\left(\cancel{2}\left(x + \frac{3\pi}{4}\right)\right) - 1$$

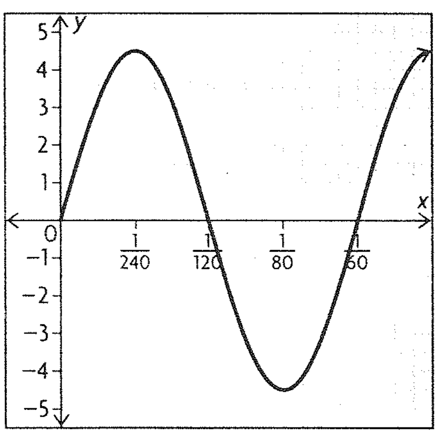
11. a) reflection in the x-axis, vertical stretch by a factor of 19, vertical translation 9 units down

b) horizontal compression by a factor of $\frac{1}{10}$, horizontal translation $\frac{\pi}{12}$ to the left

c) vertical compression by a factor of $\frac{10}{11}$, horizontal translation $\frac{\pi}{9}$ to the right, vertical translation 3 units up

d) reflection in the x-axis, reflection in the y-axis, horizontal translation π to the right

12. a)



b) period: $\frac{2\pi}{x} = \frac{120\pi}{1}$

$$(120\pi)(x) = 2\pi$$

$$x = \frac{2\pi}{120\pi}$$

$$x = \frac{1}{60}$$

c) The maximum occurs at $\left(\frac{1}{4}\right)\left(\frac{1}{60}\right) = \frac{1}{240}$.

d) The minimum occurs at $\left(\frac{3}{4}\right)\left(\frac{1}{60}\right) = \frac{3}{240} = \frac{1}{80}$.

13. a) 2π radians
b) 2π radians
c) π radians

14. a) The amplitude represents the radius of the circle in which the bumblebee is flying.

b) The period represents the time that the bumblebee takes to fly one complete circle.

c) The equation of the axis represents the height, above the ground, of the centre of the circle in which the bumblebee is flying.

d) Since the bumblebee is at it lowest point at $t = 0$, the cosine function should be used to represent the function.

15. The axis is at $\frac{15\,000 + 500}{2} = 7750$. So, the amplitude is $7750 - 500 = 7250$. The period is 12 months.

$$k = \frac{2\pi}{12} = \frac{\pi}{6}$$

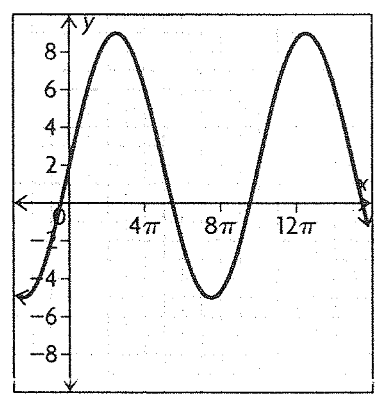
$$P(m) = 7250\cos\left(\frac{\pi}{6}m\right) + 7750$$

16. The axis is at $\frac{180 + 120}{2} = 150$. So, the amplitude is $150 - 120 = 30$. It is shifted to the right $\frac{\pi}{2}$. The period is 1.2 seconds.

$$k = \frac{2\pi}{1.2} = \frac{5\pi}{3}$$

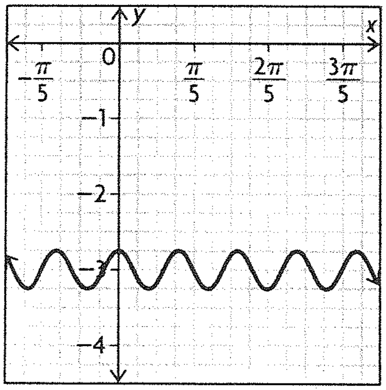
$$h(t) = 30\sin\left(\frac{5\pi}{3}t - \frac{\pi}{2}\right) + 150$$

17.



a) $0 < x < 5\pi$, $10\pi < x < 15\pi$
b) $2.5\pi < x < 7.5\pi$, $12.5\pi < x < 17.5\pi$
c) $0 < x < 2.5\pi$, $7.5\pi < x < 12.5\pi$

18.



a) $x = 0, x = \frac{1}{2}$

b) $x = \frac{1}{8}, x = \frac{5}{8}$

c) $x = \frac{3}{8}, x = \frac{7}{8}$

19. a) period: $\frac{2\pi}{x} = \frac{8\pi}{3}$

$$x = \frac{3}{4}\text{ s}$$

b) The period represents the time between one beat of a person's heart and the next beat.

c) For $t = 0.2$,

$$P(0.2) = 100 - 20\cos\left(\frac{8\pi}{3}(0.2)\right)$$

$P(0.2) = 102$

For $t = 0.3$,

$$P(0.3) = 100 - 20\cos\left(\frac{8\pi}{3}(0.3)\right)$$

$$P(0.3) \doteq 116$$

$$\frac{116 - 102}{0.3 - 0.2} = \frac{14}{0.1} = 140$$

d) For $t = 0.4$,

$$P(0.4) = 100 - 20\cos\left(\frac{8\pi}{3}(0.4)\right)$$

$P(0.4) \doteq 119.6$

For $t = 0.6$,

$$P(0.6) = 100 - 20\cos\left(\frac{8\pi}{3}(0.6)\right)$$

$$P(0.6) \doteq 93.8$$

$$\frac{93.8 - 119.6}{0.6 - 0.4} = \frac{-25.8}{0.2} = -129$$

Chapter Self-Test, p. 378

1. $y = \sec x$

2. $\sin\frac{3\pi}{2} = -1$

$\cos\pi = -1$

$\tan\frac{7\pi}{4} = -1$

$\csc\frac{3\pi}{2} = -1$

$\sec 2\pi = 1$

$\cot\frac{3\pi}{4} = -1$

$\sec 2\pi$ has a different value.

3. $y = -12\cos\left(\frac{5}{3}\left(x + \frac{\pi}{6}\right)\right) + 100$

$$y = -12\cos\left(\frac{5}{3}\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)\right) + 100$$

$y \doteq 108.5$

4. For $d = 52$ (Feb 21)

$$T(52) = -20\cos\left(\frac{2\pi}{365}(52 - 10)\right) + 25$$

$T(52) \doteq 10.0$

For $d = 128$ (May 8),

$$T(128) = -20\cos\left(\frac{2\pi}{365}(128 - 10)\right) + 25$$

$T(128) \doteq 33.9$

$$\frac{33.9 - 10.0}{128 - 52} = \frac{23.9}{76} \doteq 0.31\text{ °C per day}$$

5. $\frac{5\pi}{8}$ radians;

$$\frac{5\cancel{\pi}}{8}\ \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi\text{ radians}}}\right) = 112.5°$$

$\frac{2\pi}{3}$ radians;

$$\frac{2\cancel{\pi}}{3}\ \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi\text{ radians}}}\right) = 120°$$

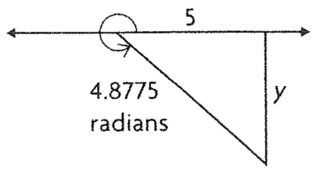
$\frac{3\pi}{5}$ radians;

$$\frac{3\cancel{\pi}}{5}\ \cancel{\text{radians}} \times \left(\frac{180°}{\cancel{\pi\text{ radians}}}\right) = 108°$$

So, from smallest to largest, the angles are $\frac{3\pi}{5}$, 110°, $\frac{5\pi}{8}$, 113°, and $\frac{2\pi}{3}$.

6. $y = \sin\left(x + \frac{5\pi}{8}\right)$

7.



$2\pi - 4.8755 = 1.4077$

$\tan(1.4077) = \frac{y}{5}$

$y = 5\tan(1.4077)$

$y \doteq -30$

8. a) $-3\cos\left(\frac{\pi}{12}x\right) + 22$

b) For $t = 0$ (sunrise)

$T(0) = -3\cos\left(\frac{\pi}{12}(0)\right) + 22$

$T(0) = 19$

For $t = 6$,

$T(6) = -3\cos\left(\frac{\pi}{12}(6)\right) + 22$

$T(6) = 22$

$\frac{22 - 19}{6 - 0} = \frac{3}{6} \doteq 0.5$ °C per hour

c) $11 \leq t \leq 13$

For $t = 11$ (5 p.m.)

$T(11) = -3\cos\left(\frac{\pi}{12}(11)\right) + 22$

$T(11) = 24.9$

For $t = 13$ (7 p.m.),

$T(13) = -3\cos\left(\frac{\pi}{12}(13)\right) + 22$

$T(13) = 24.9$

$\frac{24.9 - 24.9}{13 - 11} = \frac{0}{2} \doteq 0$ °C per hour

Chapters 4–6 Cumulative Review pp. 380–383

1.
$$\begin{aligned} x^4 + 3x^3 &= 4x^2 + 12x \\ x^4 + 3x^3 - 4x^2 - 12x &= 0 \\ (x^4 + 3x^3) + (-4x^2 - 12x) &= 0 \\ x^3(x + 3) - 4x(x + 3) &= 0 \\ (x + 3)(x^3 - 4x) &= 0 \\ x(x + 3)(x^2 - 4) &= 0 \\ x(x + 3)(x + 2)(x - 2) &= 0 \\ x &= 0, -3, -2, 2 \end{aligned}$$

The correct answer is **d)**.

2. $f(x) = a(x + 1)(x - 1)(x - 4)$

$36 = a(2 + 1)(2 - 1)(2 - 4)$

$36 = a(3)(1)(-2)$

$36 = -6a$

$-6 = a$

So, the equation is

$$\begin{aligned} f(x) &= -6(x + 1)(x - 1)(x - 4) \\ &= -6(x^2 - 1)(x - 4) \\ &= -6(x^3 - 4x^2 - x + 4) \\ &= -6x^3 + 24x^2 + 6x - 24 \end{aligned}$$

The correct answer is **b)**.

3. $2 - 3x < x - 5$

$2 - 4x < -5$

$-4x < -7$

$x > \frac{7}{4}$

-2 is not greater than $\frac{7}{4}$.

The correct answer is **a)**.

4. $-10 \leq 3x + 5 \leq 8$

$-15 \leq 3x \leq 3$

$-5 \leq x \leq 1$

The correct answer is **c)**.

5. Using the graph $f(x) < g(x)$ from $x = 2$ to ∞.

So, $x > 2$.

The correct answer is **a)**.

6. $h(t) = -5t^2 + 3.5t + 10$

$-5t^2 + 3.5t + 10 > 10$

$-5t^2 + 3.5t > 0$

$t(-5t + 3.5) > 0$

The critical points are $t = 0$ and $t = 0.7$.

Test -1: $(-1)(-5(-1) + 3.5) > 0$

$(-1)(5 + 3.5) > 0$ FALSE

Test 0.5: $(0.5)(-5(0.5) + 3.5) > 0$

$(0.5)(-2.5 + 3.5) > 0$ TRUE

Test 1: $(1)(-5(1) + 3.5) > 0$

$(1)(-5 + 3.5) > 0$ FALSE

So, the solution is $t \in (0, 0.7)$.

The correct answer is **b)**.

7. From the given information, the function must have a maximum at $x = 0$ and a minimum at $x = 2$. Choice **a)** is not a possible set of zeros for the function. If the function has zeros only at $x = 0$ and $x = 1$, then the function has to increase to $(0, 0)$ and turn there and begin to decrease. Sometime before $x = 1$, the function must turn again and increase to get back to the x-axis before $x = 1$. But this contradicts the given information that the function is decreasing for $0 < x < 2$.

The correct answer is **a)**.

8. $f(x) = 2x^3 - 4x^2 + 6x$

$f(-0.05) = 2(-0.05)^3 - 4(-0.05)^2 + 6(-0.05)$
$= -0.310\,25$

Find the rate of change using the points $(0, 0)$ and $(-0.05, -0.310\,25)$.

$m = (-0.310\,25 - 0) \div (-0.05 - 0)$
$= 6.2$

So, the correct answer is **c)**.

9. The graph has vertical asymptotes at $x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0$ and $x = 3$

Test $x = -1$: The function will be $(+) \div (-)(-) = (+)$
So, the function is positive to the left of zero.
Test $x = 1$: The function will be $(+) \div (+)(-) = (-)$
So, the function is negative between 0 and 3.
Test $x = 4$: The function will be $(+) \div (+)(+) = (+)$
So, the function is positive to the right of 3.
The function that matches this is **c)**.

10. The function will have a horizontal asymptote of 0 and vertical asymptotes of $x = -5$ and $x = 2$. So the correct answer is **c)**.

11. To have an oblique asymptote, the degree of the numerator must be greater than the degree of the denominator by exactly 1. Neither choice **a)** nor choice **c)** meets this condition. While it seems that choice **b)** meets the condition, it does not because the numerator can be factored and the function simplified. $g(x) = \frac{(x + 3)(x - 3)}{(x - 3)} = x + 3$.
The correct answer is **d)**.

12. Only functions a and b are undefined at $x = 3$. Examine the behaviour of these functions for $-2 < x < 3$.
For choice a: Test $x = 0$: $(+) \div (+) = (+)$
For choice b: Test $x = 0$: $(+) \div (-) = (-)$
So, choice a is positive for $-2 < x < 3$.
The correct answer is **a)**.

13. Choose a value that is very close to and to the left of $\frac{3}{5}$.
Try 0.599:
$(2 - 3(0.599)) \div (5(0.599) - 3) = -40.6$
So, the function approaches $-\infty$.
The correct answer is **d)**.

14. $\frac{3 - 2x}{x + 2} = 3x$

$3 - 2x = 3x(x + 2)$
$3 - 2x = 3x^2 + 6x$
$0 = 3x^2 + 8x - 3$
$0 = (3x - 1)(x + 3)$
$x = \frac{1}{3}$ and -3

The correct answer is **c)**.

15. Any of the steps listed can be used to begin solving the rational equation.
The correct answer is **d)**.

16. $2x - 3 \le \frac{2}{x}$

$$2x - 3 - \frac{2}{x} \le 0$$
$$\frac{2x^2}{x} - \frac{3x}{x} - \frac{2}{x} \le 0$$
$$\frac{2x^2 - 3x - 2}{x} \le 0$$
$$\frac{(2x + 1)(x - 2)}{x} \le 0$$

The correct answer is **a)**.

17. $x - 3 > \frac{6}{x - 2}$

$$x - 3 - \frac{6}{x - 2} > 0$$
$$\frac{x(x - 2)}{x - 2} - \frac{3(x - 2)}{x - 2} - \frac{6}{x - 2} > 0$$
$$\frac{x^2 - 2x - 3x + 6 - 6}{x - 2} > 0$$
$$\frac{x^2 - 5x}{x - 2} > 0$$
$$\frac{x(x - 5)}{x - 2} > 0$$

The critical points are 0, 5 and 2.
Test -1: $(-)(-) \div (-) = (-)$ FALSE
Test 1: $(+)(-) \div (-) = (+)$ TRUE
Test 3: $(+)(-) \div (+) = (-)$ FALSE
Test 6: $(+)(+) \div (+) = (+)$ TRUE
So, the solution is $(0, 2)$ and $(5, \infty)$.
The correct answer is **d)**.

18. Let $y = f(x)$.

$$\frac{G(a + h) - G(a)}{h} = \frac{G(1.001) - G(1)}{0.001}$$
$$= \frac{0.9985 - 1}{0.001}$$
$$\doteq -1.5$$

So, the correct answer is **b)**.

19. $\dfrac{s(a+h)-s(a)}{h}=\dfrac{s(3.001)-s(3)}{0.001}$
$=\dfrac{-7.009\,009-(-7)}{0.001}$
$\doteq -9$

So, the correct answer is **b)**.

20. $P=3\left(\dfrac{5\pi}{12}\right)+2(3)$
$=\dfrac{5\pi}{4}+6$ m

The correct answer is **b)**.

21. $20\left(\dfrac{\pi}{180}\right)=\dfrac{\pi}{9}$
$135\left(\dfrac{\pi}{180}\right)=\dfrac{3\pi}{4}$
$-270\left(\dfrac{\pi}{180}\right)=-\dfrac{3\pi}{2}$

Each of the pairs of angles are equivalent.
The correct answer is **d)**.

22. $\tan^{-1}\left(\dfrac{7}{4}\right)\doteq 1.0517$
$\pi-1.0517\doteq 2.09$

The correct answer is **c)**.

23. Since $\sin\theta=-\frac{\sqrt{3}}{2}$, the value for r is 2 and the value for $a=\sqrt{3}$.
So, find b.
$(\sqrt{3})^2+b^2=2^2$
$3+b^2=4$
$b^2=1$
$b=1$

The angle is in either in the third or fourth quadrant since the sine is negative.
So, $\cos\theta=\frac{1}{2}$ or $-\frac{1}{2}$.
$\tan\theta=\sqrt{3}$ when $\cos\theta=-\frac{1}{2}$
or $-\sqrt{3}$ when $\cos\theta=\frac{1}{2}$

The correct answer is **a)**.

24. $x=\sin^{-1}0.5$
$x=\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$

The correct answer is **d)**.

25. The graph shown has been stretched vertically by a factor of 3, since the minimum is -4 and the maximum is 2, a difference of 6. The graph has also been compressed horizontally by a factor of $\frac{1}{2}$, as the period shown is only π. The graph has been translated down 1 unit. The equation for the graph shown is $y=3\sin(2x)-1$.
The correct answer is **b)**.

26. The equation shows a horizontal stretch by a factor of 3 and a horizontal translation of 2π units to the left.
The correct answer is **d)**.

27. The maximum height represented by choice **b)** is $41-5=36$, not 41 as required.
The maximum height represented by choice **c)** is $18-23=-5$, not 41 as required.
The maximum height represented by choice **d)** is $41-36=5$, not 41 as required.
The maximum height represented by choice **a)** is $18+23=41$, as required. The minimum height represented by choice **a)** is $-18+23=5$, as required.
The correct answer is **a)**.

28. Using a graphing calculator, the function is decreasing in both intervals which means the instantaneous rate of change is negative in both intervals.
The correct answer is **c)**.

29. $P(0)=23.7\cos\left(\dfrac{\pi}{6}(0-7)+24.1\right)$
$\doteq 3.575$

$P(4)=23.7\cos\left(\dfrac{\pi}{6}(4-7)+24.1\right)$
$\doteq 24.1$

So, the rate of change is $(24.1-3.575)\div(4-0)$ or 5.131 25.

$P(1)=23.7\cos\left(\dfrac{\pi}{6}(1-7)\right)+24.1$
$\doteq 0.4$

$P(7)=23.7\cos\left(\dfrac{\pi}{6}(0)\right)+24.1$
$\doteq 47.8$

So, the rate of change is $(47.8-0.4)\div(7-1)$ or 7.9.

$P(16)=23.7\cos\left(\dfrac{\pi}{6}(16-7)\right)+24.1$
$\doteq 24.1$

So, the rate of change is $(24.1-47.8)\div(16-7)$ or -2.63.

$P(10)=23.7\cos\left(\dfrac{\pi}{6}(10-7)\right)+24.1$
$\doteq 24.1$

$$P(18) = 23.7\cos\left(\frac{\pi}{6}(18-7)\right) + 24.1$$
$$\doteq 44.62$$

So, the rate of change is $(44.62 - 24.1) \div (18 - 10)$ or 2.565.

The correct answer is **b)**.

30. a) length $= 50 - 2x$
width $= 40 - 2x$
height $= x$

$$V = lwh$$
$$= (50 - 2x)(40 - 2x)x$$

b)
$$6000 = (50 - 2x)(40 - 2x)x$$
$$6000 = (2000 - 80x - 100x + 4x^2)x$$
$$6000 = 2000x - 180x^2 + 4x^3$$
$$0 = 4x^3 - 180x^2 + 2000x - 6000$$
$$0 = x^3 - 45x^2 + 500x - 1500$$

The possible reasonable solutions are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20$.
Use synthetic division to determine the solution.

$$\begin{array}{r|rrrr} 3 & 1 & -45 & 500 & -1500 \\ & \downarrow & 3 & -126 & 1122 \\ \hline & 1 & -42 & 374 & -378 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -45 & 500 & -1500 \\ & \downarrow & 5 & -200 & 1500 \\ \hline & 1 & -40 & 300 & 0 \end{array}$$

So, $x = 5$.
Now solve $x^2 - 40x + 300 = 0$
$(x - 10)(x - 30) = 0$
So, $x = 10$ or 30, but 30 does not make sense in the context of the problem. So, $x = 5$ or 10.

c) Using a graphing calculator, find the relative maximum value. It occurs when $x \doteq 7.4$ cm.

d) Using a graphing calculator, the range is $3 < x < 12.8$.

31. a) $f(x)$: $0 = x^2 - 5x + 6$
$$0 = (x - 3)(x - 2)$$
$$x = 3 \text{ and } x = 2$$
$g(x)$: $0 = x - 3$
$$x = 3$$
$\dfrac{f(x)}{g(x)}$: $0 = \dfrac{x^2 - 5x + 6}{x - 3}$

The function is undefined at $x = 3$, so the zero is $x = 2$.

$\dfrac{g(x)}{f(x)}$: $0 = \dfrac{x - 3}{x^2 - 5x + 6}$

The function is undefined at 2 and 3, so there are no zeros.

b) $\dfrac{f(x)}{g(x)}$: hole at $x = 3$, no asymptotes

$\dfrac{g(x)}{f(x)}$: hole at $x = 3$, asymptotes at $x = 2$ and $y = 0$

c) tangent at $x = 1$

$\dfrac{f(x)}{g(x)}$: $y = x - 2$

$\dfrac{g(x)}{f(x)}$: $y = -x$

32. a) Vertical compressions and stretches do not affect location of zeros; maximum and minimum values are multiplied by the scale factor, but locations are unchanged; instantaneous rates of change are multiplied by the scale factor; Horizontal compressions and stretches move locations of zeros, maximums, and minimums toward or away from the y-axis by the reciprocal of the scale factor; instantaneous rates of change are multiplied by the reciprocal of scale factor; Vertical translations change location of zeros or remove them; maximum and minimum values are increased or decreased by the amount of the translation, but locations are unchanged; instantaneous rates of change are unchanged; Horizontal translations move location of zeros by the same amount as the translation; maximum and minimum values are unchanged, but locations are moved by the same amount as the translation; instantaneous rates of change are unchanged, but locations are moved by the same amount as the translation.

b) $y = \cos x$: Vertical compressions and stretches do not affect location of zeros; maximum and minimum values are multiplied by the scale factor, but locations are unchanged; instantaneous rates of change are multiplied by the scale factor; Horizontal compressions and stretches move locations of zeros, maximums, and minimums toward or away from the y-axis by the reciprocal of the scale factor; instantaneous rates of change are multiplied by the reciprocal of scale factor; Vertical translations change location of zeros or remove them; maximum and minimum values are increased or decreased by the amount of the translation, but locations are unchanged; instantaneous rates of change are unchanged; Horizontal translations move location of zeros by the same amount as the translation; maximum and minimum values are unchanged, but locations are moved by the same amount as the translation; instantaneous rates of change are

unchanged, but locations are moved by the same amount as the translation.

$y = \tan x$: Vertical compressions and stretches do not affect location of zeros; instantaneous rates of change are multiplied by the scale factor; Horizontal compressions and stretches move locations of zeros toward or away from the y-axis by the reciprocal of the scale factor; instantaneous rates of change are multiplied by the reciprocal of scale factor; Vertical translations change location of zeros or remove them; instantaneous rates of change are unchanged; Horizontal translations move location of zeros by the same amount as the translation; instantaneous rates of change are unchanged, but locations are moved by the same amount as the translation.